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On the condition for the hyperbolicity of systems with double characteristic roots, II

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In this paper, we consider the Cauchy problem of systems with double characteristics;

$$\begin{cases} (1) \quad Pu \equiv D_t u + \sum_{i=1}^{n} A^i(t, x) D_{x_i} u + Bu = f, \qquad \text{in } \Omega \subset \boldsymbol{R}_t \times \boldsymbol{R}_x^n, \\ (2) \quad u(t_\circ, x) = u_\circ(x), \end{cases}$$

where $A^i(t, x)$ and $B(t, x)$ are of order N and $A^i(t, x)$ are real $(1 \leqq i \leqq n)$. We assume the following:

Assumption 1. Each characteristic root $\tau = \lambda_j(t, x; \xi)$ of $\det P_p(t, x; \tau, \xi) = 0$ is real, of constant multiplicity and at most double.

Let λ_j be double for $1 \leqq j \leqq r$ and be simple for $r+1 \leqq j \leqq s$. As well known, under the assumption 1, the following condition (L) is necessary for the $\mathcal{E}$ well-posedness.

$$\text{(L)} \qquad {}^{co}P_p P_s {}^{co}P_p + \frac{1}{2i} {}^{co}P_p \{P_p, {}^{co}P_p\} |_{\tau=\lambda_j} \equiv 0, \qquad (1 \leqq j \leqq r).$$

Here, P_p, P_s and ${}^{co}P_p$ are the principal symbol of P, the subprincipal symbol of P and the cofactor matrix of P_p. In the previous paper, we considered 1) the consequence of the condition (L), 2) the existence of stably non-hyperbolic operators with only real characteristic roots, 3) sufficients conditions for the $\mathcal{E}$ well-posedness, restricting ourselves to the case of $n=1$. In this article, we shall generalize the results in the sections 1 and 3 in [26]. However, there exists some difficulties proper to the case in higher dimension domains. Therefore the results in this article are a little rougher than those in [26].

We shall use the notation and the definitions given in [26] without mention of it. We shall name the sections in the previous paper [26] and this paper with the straight numbers. Then, we shall start from the section 4. (If we say "the section 1", it means "the section 1 in [26]".)