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A class of imperfect prime ideals having the equality of ordinary and symbolic powers

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1. Introduction.

Given a polynomial ring R over a field, we are interested in prime ideals $\mathfrak{p}\subset R$ having the following property:
(A) $\mathfrak{p}^{n}=\mathfrak{p}^{(n)}$ for every positive integer n, where $\mathfrak{p}^{(n)}$ denotes the n-th symbolic power of $\mathfrak{p}$, i.e. the $\mathfrak{p}$-primary component of $\mathfrak{p}^{n}$.

In [5, Theorem 1], Hochster proved that (A) is equivalent to each of the following properties:

(B) $gr_{\mathfrak{p}}(R):=\bigoplus_{n=0}^{\infty}\mathfrak{p}^{n}/\mathfrak{p}^{n+1}$, the associated graded ring of R with respect to $\mathfrak{p}$, is a domain.
(C) The Rees ring $R[T, \mathfrak{p}T^{-1}]$, the subring of $R[T, T^{-1}]$ generated over R by the indeterminate T and the elements aT^{-1} with $a\in\mathfrak{p}$, is a unique factorization domain.

On the other hand, Samuel had conjectured that a unique factorization domain is a Cohen-Macaulay ring. Thus, it may be possible that (A) or (B) implies the Cohen-Macaulay property of $gr_{\mathfrak{p}}(R)$ because, by [6, Theorem 4.11], the Cohen-Macaulay property of $gr_{\mathfrak{p}}(R)$ is equivalent to the Cohen-Macaulay property of $R[T, \mathfrak{p}T^{-1}]$. If we have a prime ideal $\mathfrak{p}\subset R$ with (A) then we can construct either a Cohen-Macaulay graded domain or a counter-example to Samuel's conjecture.

Until now, beside some solitary examples, only two classes of prime ideals $\mathfrak{p}$ with (A) in polynomial rings over a field have been known:
1) $\mathfrak{p}$ is a complete intersection prime (see, e.g., [5, (2.1)]).
2) $\mathfrak{p}$ is generated by the $r\times r$ minors of an $r\times s$ matrix of indeterminates, $r\geqq s$ (see [5, (2.2)], [14] or [2]).

By all known prime ideals $\mathfrak{p}$ with (A) $gr_{\mathfrak{p}}(R)$ is always a Choen-Macaulay domain. Note that Nagata had raised the question of whether the zero-graded part of a positively graded Cohen-Macaulay ring is a Cohen-Macaulay ring [10, Question 3]. So one might also expect that (A) implies the Cohen-Macaulay property of $R/\mathfrak{p}$, the zero-graded part of $gr_{\mathfrak{p}}(R)$. But, like Nagata's question which was negatively answered in [10], that is not true. The first counter-example for that was shown by Hochster [5, (2.3)], and an another can be found