J. Math. Kyoto Univ. (JMKYAZ)
21-3 (1981) 547-574

Leray-Volevich's system and Gevrey class

By

Kunihiko KAJITANI

(Communicated by Prof. S. Mizohata, April 14, 1980)

§1. Introduction

We consider the Cauchy problem for hyperbolic systems with multiple characteristics of constant multiplicity. Let Ω be a band $[0, T]\times R^n$ in R^{n+1}. We consider the following equations in Ω,

$$\sum_{q=1}^{N} a_q^p(x, D)u^q(x)=f^p(x), \qquad p=1, \cdots, N, \tag{1.1}$$

where $x=(x_0, x_1, \cdots, x_n)=(x_0, x')\in\Omega$ and $a_q^p(x, D)$ differential operators of order m_q^p of which coefficients are in the Gevrey class $\gamma^s(\Omega)(s\geqq 1)$.

We use the notation as follows,

$$D=(D_0, \cdots, D_n), \qquad D_k=-\sqrt{-1}\frac{\partial}{\partial x_k},$$

$$\alpha=(\alpha_0, \cdots, \alpha_n), \qquad \alpha_k \text{ integers},$$

$$D^\alpha=D_0^{\alpha_0}D_1^{\alpha_1}\cdots D_n^{\alpha_n}, \qquad |\alpha|=\sum\alpha_k,$$

$$\xi=(\xi_0, \xi_1, \cdots, \xi_n); \text{ dual variables of } x,$$

and $\gamma_s(\Omega)$ consists of all functions f such that there exists positive constants C and A satisfying for any α,

$$|D^\alpha f(x)|\leqq CA^{|\alpha|}|\alpha|!^s, \qquad x\in\Omega.$$

We correspond the polynomial $a_q^p(x, \xi)$ in ξ to a differential operator $a_q^p(x, D)$. We denote by $\mathring{a}_q^p(x, \xi)$ the homegeneous part of degree m_q^p of $a_q^p(x, \xi)$. We define the total order m of $\{a_q^p(x, D)\}$ such that

$$m=\max_\pi \sum_{p=1}^{N} m_{\pi(p)}^p,$$

where π runs over all permutations of $[1, \cdots, N]$. Then it follows from Volevich's lemma [16] that there exists a pair of integes $\{t_p, s_p\}$, $p=1, \cdots, N$, such that

$$\begin{aligned} &m_q^p\leqq t_q-s_p, \qquad (p, q)\in[1, \cdots, N]^2, \\ &m=\sum_{p=1}^{N}(t_p-s_p), \end{aligned} \tag{1.2}$$