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On the equations of one-dimensional motion of compressible viscous fluids

By

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§ 1. Introduction

We consider the equations of the one-dimensional motion of a compressible, viscous and heat-conductive fluid in Lagrangian coordinates:

$$\begin{cases} \rho_t+\rho^2 u_x=0, \\ u_t+p_x=(\mu\rho u_x)_x, \\ \theta_t+\dfrac{\theta p_\theta}{c_V}u_x=\dfrac{1}{c_V}\{(\kappa\rho\theta_x)_x+\mu\rho u_x^2\}, \end{cases} \tag{1.1}$$

where t is time and x denotes the Lagrangian mass coordinate. Here the unknown functions ρ, u and θ represent the density, velocity and absolute temperature of the fluid; the pressure p and the heat capacity at constant volume c_V are related to the thermodynamic quantities $\rho>0$ and $\theta>0$ by the equations of state, and p_θ denotes $\partial p/\partial\theta$; μ and κ are the coefficients of viscosity and heat conduction respectively.

We assume the following conditions on the system (1.1).

A_1: $p=p(\rho, \theta)$ and $c_V=c_V(\rho, \theta)$ are smooth functions of $(\rho, \theta)\in\mathscr{D}_{\rho,\theta}\equiv\{\rho>0, \theta>0\}$ and satisfy the general equations of state on $\mathscr{D}_{\rho,\theta}$, that is,

$$p_\rho\equiv\frac{\partial p}{\partial\rho}>0, \quad c_V>0, \quad \frac{\partial c_V}{\partial\rho}=-\theta p_{\theta\theta}/\rho^2,$$

where $p_{\theta\theta}=\partial^2 p/\partial\theta^2$.

A_2: $\mu=\mu(\rho)$ and $\kappa=\kappa(\rho)$ are smooth functions of $\rho>0$ (independent of $\theta>0$) and satisfy $\mu>0$ and $\kappa>0$ for $\rho>0$.

The assumptions $p_\rho>0$ and $c_V>0$ imply that the system (1.1) with $\mu=\kappa=0$ is hyperbolic, while the assumptions $\mu>0$ and $\kappa>0$ imply that the equations of u and θ in (1.1) are parabolic.