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Sobolev spaces over the Wiener space based on an Ornstein-Uhlenbeck operator

By

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1. Introduction

Sobolev spaces over the Wiener space, or more generally over an abstract Wiener space, play a fundamental role in the Malliavin calculus. They are based on an *Ornstein-Uhlenbeck operator*, denoted by L, on the Wiener space. In this paper, we consider a different kind of Ornstein-Uhlenbeck operator L_A.

To be precise, let (B, H, μ) be an abstract Wiener space, i.e. B is a real separable Banach space and μ is a Gaussian measure with a reproducing kernel Hilbert space H. Let A be a strictly positive definite self-adjoint operator in H. We consider the following semigroup (called an Ornstein-Uhlenbeck semigroup)

$$T_t f(x)=\int_B f(e^{-tA}x+\sqrt{1-e^{-2tA}}y)\mu(dy). \tag{1.1}$$

The generator of $\{T_t\}$ is an operator in our consideration. We denote it by L_A. Moreover the associated Dirichlet form is given by

$$\mathcal{E}(f, g)=\int_B(\sqrt{A^*}Df(x), \sqrt{A^*}Dg(x))_{H^*}\mu(dx) \tag{1.2}$$

where D is the H-derivative. If $A=1$, then L_A is the usual Ornstein-Uhlenbeck operator L.

Its origin is in the quantum field theory. In physical literature, the Ornstein-Uhlenbeck operator in the Malliavin calculus is called the *Number operator* and our Ornstein-Uhlenbeck operator is the *free Hamiltonian*. So this operator is important in physics.

To construct Sobolev spaces in the Malliavin calculus, the Meyer equivalence which insists the equivalence of two kinds of norms defined by L and D, is essential. Such a problem was first discussed by P.A. Meyer [12] and then M. Krée, P. Krée [9] and H. Sugita [22] proved it in the higher degree case. In this paper, we will obtain an analogous equivalence. This problem was proposed by J. Potthoff [16]. The equivalence in our case is a little bit different from that in the Malliavin calculus because A is not bounded in general.

The organization of this paper is as follows. In the section 2, we give a precise

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