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Semi-Lévy processes, semi-selfsimilar additive processes, and semi-stationary Ornstein-Uhlenbeck type processes

By

Makoto Maejima and Ken-iti Sato

1. Introduction

The works of Wolfe [27], Jurek and Vervaat [6], Sato and Yamazato [20], [21], Sato [16], and Jeanblanc, Pitman, and Yor [4] combined show that the following three classes have one-one correspondence with each other — the class of selfsimilar additive processes, the class of stationary Ornstein-Uhlenbeck type processes, and the class of homogeneous independently scattered random measures (Lévy processes) with finite log-moment. The correspondence is given by Lamperti transformations and stochastic integrals. This correspondence gives representations of the class of selfdecomposable distributions. The aim of this paper is to give extensions of this correspondence to certain wider classes and to discuss Ornstein-Uhlenbeck type processes in a wide sense.

There are two significant classes that extend the class of stable distributions — the class of selfdecomposable distributions and the class of semi-stable distributions. The class of semi-selfdecomposable distributions is a natural extension of these two classes (see [9]). Their description in terms of Lévy measures is given in [17]. Thus we are motivated to generalize the representations of the class of selfdecomposable distributions to those of the class of semi-selfdecomposable distributions. In the case of distributions on $\mathbb{R}^d$ with $d \geqslant 2$, we will simultaneously deal with another sort of generalization. This is related to Q-stable and Q-selfdecomposable distributions (see [21]), Q-selfsimilar additive processes (see [16]), and Ornstein-Uhlenbeck type processes with drift $-Qx$ (see [19], [21], [26]), where Q is a $d \times d$ matrix in $\mathbf{M}_d^+$ defined below.

Before going to statement of main results, let us give some definitions.

Let $\mathbf{M}_d$ be the class of $d \times d$ real matrices and $\mathbf{M}_d^+$ the class of $Q \in \mathbf{M}_d$ all of whose eigenvalues have positive real parts. Let I be the identity matrix and $a^Q = \sum_{n=0}^{\infty}(n!)^{-1}(\log a)^n Q^n \in \mathbf{M}_d$ for $a > 0$ and $Q \in \mathbf{M}_d$. Sometimes we also use the class $\mathbf{M}_{l\times d}$ of $l \times d$ real matrices. Denote the transpose of $F \in \mathbf{M}_{l\times d}$ by F'. Let $\mathcal{L}(X)$ be the distribution of a random element X. When $\mathcal{L}(X) = \mathcal{L}(Y)$ for two random elements X and Y, we write $X \overset{\mathrm{d}}{=} Y$. For

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