J. Math. Kyoto Univ. (JMKYAZ)
44-3 (2004), 679–684

On the Stiefel-Whitney classes of the adjoint representation of E_8

By

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Introduction

Exceptional Lie groups G_2, F_4 and $E_l\,(l = 6, 7, 8)$ have been studied by many topologists, where the subscript refers to the rank and we agree to consider 1-connected and compact ones tacitly. The cohomology of the classifying space of them is determined to a large extent. The mod 2 cohomology of BE_8, however, is left unknown. The ring structure of that of BE_7 is not determined yet.

It is known classically that an elementary abelian 2-subgroup, a 2-torus in other words, of the maximal rank is useful. This rank is called the 2-rank of the Lie group. Note that a maximal 2-torus does not necessarily give the 2-rank (see [1], [11]). On the other hand, the 3-connected covering $\widetilde{E}_l$ of E_l has been also utilized. In this paper we determine the image of the Stiefel-Whitney classes of the adjoint representaion of E_8 in $H^*(B\widetilde{E}_8; \boldsymbol{F}_2)$. In particular, we give some results on the image of $H^*(BE_8; \boldsymbol{F}_2)$ in it. We denote the mod 2 cohomology of X simply by $H^*(X)$ and by A^* the mod 2 Steenrod algebra. If S is a non-empty subset of an algebra, $\langle S\rangle$ denotes the subalgebra generated by S.

The author is very grateful to Professor Akira Kono for his helpful advices during the preparation of this paper.

1. Cohomology of the classifying spaces of 3-connected cover

First we recall here facts related to BE_l for later use. Let T^l be a maximal torus of E_l. Denote by q' a generator of $H^4(BE_l; \boldsymbol{Z})$ and by q'' the induced map defined on BT^l. Let $B\widetilde{E}_l$ and $B\widetilde{T}^l$ be the homotopy fibres of these maps, respectively. We have the natural maps $\lambda_l : BT^l \to BE_l$, $\widetilde{\lambda}_l : B\widetilde{T}^l \to B\widetilde{E}_l$, $\pi_l : B\widetilde{E}_l \to BE_l$, and $\widehat{\pi}_l : B\widetilde{T}^l \to BT^l$. Let us denote by φ_l and $\widetilde{\varphi}_l$ the natural maps $BE_{l-1} \to BE_l$ and $B\widetilde{E}_{l-1} \to B\widetilde{E}_l$, respectively. The following diagrams are commutative.

Received September 24, 2004