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A sequence of blowing-ups connecting moduli of sheaves and the Donaldson polynomial under change of polarization

By

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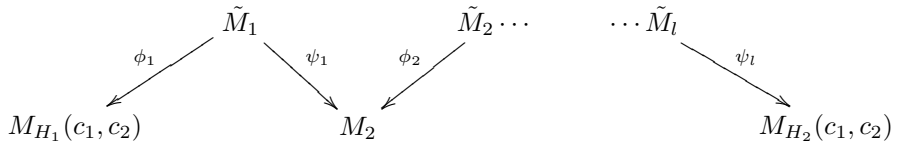
Introduction

Let X be a nonsingular projective surface over $\mathbb{C}$, H an ample line bundle on X, and $M_H(c_1, c_2)$ the moduli scheme of S-equivalence classes of rank-two H-semistable sheaves on X with fixed Chern classes (c_1, c_2). It is projective over $\mathbb{C}$.

Fix two ample line bundles H_1 and H_2 on X. In this article, we connect $M_{H_1}(c_1, c_2)$ with $M_{H_2}(c_1, c_2)$ by a sequence of blowing-ups and blowing-downs

(0.1)



using canonical properties of moduli schemes, and study the exceptional divisor E_i of ϕ_i in (0.1). Further, we apply this sequence to the calculation of the Donaldson polynomial of X. We shall algebro-geometrically inquire into the fact the Donaldson polynomials of X are independent of the choice of Riemannian metrics when $b_2^+(X) = 2p_g(X) + 1 > 1$.

Now let us survey the historical background and outline the content of this article. Roughly speaking, two methods have been developed to describe the change of moduli of sheaves under the change of polarization as a sequence of (birational) morphisms. First, Matsuki and Wentworth [MW] succeeded in connecting $M_{H_1}(c_1, c_2)$ and $M_{H_2}(c_1, c_2)$ by a sequence of Thaddeus-type flips. They introduced the notion of twisted stability of sheaves, and reduced the construction of the flip (0.1) to the Mumford-Thaddeus principle, which dealt with the change of GIT quotients under a variation of G-linearization.

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