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ABEL SUMMABILITY OF THE AUTOREGRESSIVE SERIES FOR THE BEST LINEAR LEAST SQUARES PREDICTORS

M. L. HUANG, R. A. KERMAN AND Y. WEIT

1. Introduction

Let $(\Omega, \mathcal{F}, P)$ be a probability space. Denote by $L^2(\Omega, \mathcal{F}, P)$ the Hilbert space of complex-valued random variables X, with expectation $E(X) = \int_\Omega X\,dP = 0$ and variance $E(|X|^2) < \infty$, having inner product $\langle X, Y\rangle = E(X\overline{Y})$. A sequence of random variables $\{X_k\}_{k=-\infty}^{\infty}$ in $L^2(\Omega, \mathcal{F}, P)$ is a weakly stationary stochastic process (WSSP) if for all $k,\ l \in Z$, the second moment $E(X_k\overline{X}_{k+l})$ depends only on l. The covariance function $K(l) = E(X_k\overline{X}_{k+l})$ thus defined is nonnegative definite and so

$$K(l) = \int_T e^{-il\theta}\,dF(e^{i\theta}), \qquad l \in Z \tag{1.1}$$

for an essentially unique function F,which is bounded and nondecreasing in θ on $T = [-\pi, \pi)$.

Given $n \geq 1$, the best linear least squares predictor of X_n, based on past and present observations, is defined to be the orthogonal projection of X_n on $M = \overline{sp}\{X_k, k \leq 0\}$, the closed linear span of $\ldots, X_{-2}, X_{-1}, X_0$. The projection is denoted by $\widehat{X}_n$. We assume the WSSP $\{X_k\}_{k=-\infty}^{\infty}$ is purely nondeterministic, in the sense that $\cap_{m=0}^{\infty}\overline{sp}\{X_k, k \leq -m\} = \{0\}$. This guarantees that $X_n \notin M$ for all $n \geq 1$ and that the function F in (1.1) is absolutely continuous with respect to Lebesgue measure on $T, dF(e^{i\theta}) = w(e^{i\theta})d\theta$. Moreover, the function $w(e^{i\theta})$, the spectral density of the WSSP, can be expressed in the form $w(e^{i\theta}) = |\phi(e^{i\theta})|^2$, where the so-called optimal factor $\phi = \phi(z)$ is an outer function in the Hardy space $H^2(D)$ on the unit disk $D = \{z \in \mathbb{C}\colon |z| < 1\}$ and has no zero in D.[1] See [5, pp. 53, 69].

The Spectral Theorem for unitary operators yields a Hilbert space isomorphism between the time domain $L^2(\Omega, \mathcal{F}, P)$ and the spectral domain

$$L^2(w) = \left\{ f\colon \|f\|_2 := \left[\int_T |f(e^{i\theta})|^2 w(e^{i\theta})\,d\theta\right]^{1/2} < \infty \right\},$$

in which $X_k \longleftrightarrow e^{-ik\theta}$, $k = 0, \pm1, \pm2, \ldots$; see [2, p. 241]. Denote by ϕ_n the image of $\widehat{X}_n, n \geq 1$, under the isomorphism, so that ϕ_n is the projection of $e^{-in\theta}$

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[1] Since $\phi(0) \neq 0$, we may assume, without loss of generality, that $\phi(0) = 1$.