ON 2-GROUPS OPERATING ON PROJECTIVE PLANES[1]

BY

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1. Introduction

Let $\mathbf{E}$ be a projective plane of finite order n and G a group of automorphisms of $\mathbf{E}$, whose order is a power of 2. If $n \equiv 3 \pmod 4$, then the structure of G can be determined completely. This has been done in [5, Satz 1]. Here we consider the cases $n \equiv 5 \pmod 8$ and $n \equiv 1 \pmod 8$. If $n \equiv 5 \pmod 8$, we can again give a complete list of all possible isomorphism types (see Theorem 5). They are precisely those which also occur in the automorphism groups of desarguesian planes of the corresponding order (provided, of course, that such planes exist). If $n \equiv 1 \pmod 8$, this ceases to be true, and apparently it becomes quite difficult to describe all possibilities. However in many situations one still can determine the structure of the Sylow-2-subgroups of all composition factors of a given group of automorphisms of $\mathbf{E}$ (see Theorems 2 and 4).

2. Notations and definitions

Our notation is standard except perhaps in the following abbreviations:

Z_n the cyclic group of order n

D_n the dihedral group of order n

Q_n the (generalized) quaternion group of order n

M_{2^m} the group with generators a and b and relations $a^{2^{m-1}} = b^2 = 1$, $bab = a^{1+2^{m-2}}$

$\mathbf{Z}G$ the center of the group G

$\mathbf{C}_G H$ the centralizer of the subgroup H in the group G

$\mathbf{N}_G H$ the normalizer of the subgroup H in the group G

$G_{P_1,\cdots,P_n}$ the stabilizer of the permutation group G on the points $P_1, \cdots, P_n$

PQ the line containing the points P and Q

$\Omega_1(G) = \langle g \mid g \in G \text{ and } g^2 = 1\rangle$

$\mho_1(G) = \langle g^2 \mid g \in G\rangle$

$x \circ y = x^{-1}y^{-1}xy$

$x^y = y^{-1}xy$

$G(P, g)$ the subgroup of the automorphism group G consisting of all perspectivities with center P and axis g

$n \mid m$ n divides m

$2^n \parallel m$ 2^n divides m, but 2^{n+1} does not divide m

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