MATERIAL SYMMETRY RESTRICTIONS FOR CERTAIN LOCALLY COMPACT SYMMETRY GROUPS[1]

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Part I: Preliminaries

1. Introduction. The response functions or functionals which appear in constitutive equations may all be symbolized by $y = \Phi(x)$, where x and y are tensors, or aggregates of tensors, and may thus be considered vectors in abstract real vector spaces X and Y respectively. The material symmetries of such a system may then all be described by a group G, which has representations S and T by invertible linear operators in X and Y respectively. A central problem in the formulation of constitutive equations is to find the canonical forms of form-invariant, and thus physically admissible, functions Φ—that is, functions Φ which satisfy $T(g)\Phi(x) = \Phi(S(g)x)$ for all g in G.

The standard techniques for this problem have been developed largely by Rivlin and his co-workers, [7], [11], and ref.'s in [14], but often require the assumption that Φ is a polynomial. All assumptions on the form of Φ have been removed by Wineman and Pipkin, first for finite symmetry groups [9] and then for compact symmetry groups [14]. This paper extends the conclusions of Wineman and Pipkin to a large class of locally compact groups, namely Lindelof mean-ergodic groups, through the use of a more general concept of the group average. That is, the now possible non-compactness of G requires that we first prove a topological ergodic theorem, based on the mean ergodic theorem of Calderón [2], and then use this to find as before, the restrictions on form-invariant functions.

In the arguments of Wineman and Pipkin, the invariant Hurwitz integral on a compact group [12] is used repeatedly to take averages over the group G. In our more general discussion, the left (or right) invariant Haar integral [6] is required for this purpose; but this integral cannot be used uncritically to compute group averages, for a non-compact group has infinite Haar measure. To obtain group averages, we require a theorem stating the convergence of averages on larger and larger compact open subsets of G—that is, a topological ergodic theorem in the style of the ergodic theorems for various norms [7]. Our first task is therefore to prove Theorem 4, which will be the principal tool in this paper, but readers who wish to avoid such details may skip to Part II, and read only the statement of this theorem.

2. Integration of vector-valued functions. Over the group G we shall wish to integrate not only functions $\varphi(g)$ on G alone, but also functions $\Phi(g, x)$ on

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