IMPROVING THE INTERSECTION OF POLYHEDRA IN 3-MANIFOLDS

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1. Introduction

Bing showed [10] that for a pair of intersecting 2-spheres S and S' in E^3 such that S' is tame there is a small homeomorphism of E^3 onto itself which adjusts S so that the components of its intersection with S' consist of a finite number of mutually exclusive simple closed curves in the inaccessible part of a Sierpinski curve together with sets of small diameter which fail to intersect the Sierpinski curve. Theorems 6.1 and 6.2 of this paper show that analogous results hold for topological embeddings of polyhedra in 3-manifolds. In order to prove these theorems we will need to extend to the case of polyhedra certain results about tame Sierpinski curves on spheres. These results were developed by Bing in [5]–[9].

In general we follow the definitions employed in [1]–[10]. We include a few important old definitions here as well as introduce a few new terms.

We use the term *complex* to mean *geometric complex* and we allow infinite complexes. Simplexes are closed. That is, they contain their boundaries.

An *n-manifold* is a separable metric space such that each point has a neighborhood which is homeomorphic to Euclidean n-space E^n. An *n-manifold with boundary* is a separable metric space such that each point has a neighborhood which is homeomorphic to either Euclidean n-space or the closed upper half space of Euclidean n-space. We use the term *surface* as a synonym for 2-manifold with boundary.

In a 3-manifold a set X which is homeomorphic to a polyhedron is *tame* if there is a triangulation of the manifold in which X underlies a subcomplex. For triangulated 3-manifolds an equivalent definition is that there is a homeomorphism of the 3-manifold onto itself which carries X onto a polyhedron [1], [22]. A set X in a 3-manifold is *locally tame at a point p* if there is a neighborhood N of p in the 3-manifold and a homeomorphism of Cl (N) into a 3-simplex which takes Cl $(N) \cap X$ onto a polyhedron. A set X in a 3-manifold is *locally tame* if it is locally tame at each of its points. In [1], [22] it is shown that if a closed subset X of a triangulated 3-manifold is locally tame then there is a homeomorphism of the 3-manifold onto itself which carries X onto a polyhedron.

An arc ab in E^3 *pierces* a disk D at a point p of Int (ab) if there is a neighborhood of p in ab which intersects D only at p and a positive number ε such

Received March 28, 1967.

[1] This material composes a portion of the author's Ph.D. thesis at the University of Wisconsin which was written under the supervision of Professor R. H. Bing. Work on the material was supported by National Science Foundation contracts.