EXTENSIONS AND RETRACTIONS OF ALGEBRAS OF CONTINUOUS FUNCTIONS

BY

C. W. KOHLS AND L. J. LARDY[1]

1. Introduction

A partial duality for homomorphisms between matrix algebras of continuous functions into the real numbers, the complex numbers, or the integers modulo a prime is obtained in §2. This leads to results on retractions and splitting extensions of algebras of continuous functions. We then present, for homomorphisms between matrix algebras of continuous real- or complex-valued functions, a representation by means of a continuous mapping between the underlying spaces and an automorphism. This is used in §3 to obtain a natural bijection from the family of equivalence classes of extensions of a matrix algebra of continuous functions by another to a subset of the product set of continuous mappings of certain related spaces and quotient groups of groups of units in related algebras. Finally, we give conditions under which an extension of an algebra of real- or complex-valued functions by another consists of functions that are continuous with respect to a topology obtained naturally from the topologies on the original spaces (cf. [9, Corollary 1]).

For background the reader is referred to [3] and [10]. We shall use the notation, terminology, and results of [3] and [10] freely.

In this paper, the real field, complex field and field of integers modulo a prime p are denoted by R, K, and J_p, respectively. The letter $\mathcal{K}$ will be a generic symbol for R, K, or J_p; after Corollary 1, it will indicate R or K only. If the $\mathcal{K}$-algebra A has an identity, the element $1E_{ij}$ of $L_n(A)$ is written simply E_{ij}, and the identity matrix is designated by I. We do *not* assume that a $\mathcal{K}$-homomorphism between $\mathcal{K}$-algebras with identity necessarily maps one identity to the other.

All topological spaces are assumed to be completely regular Hausdorff spaces. The algebra of all continuous functions from a space X into $L_n(\mathcal{K})$ will be denoted by $C_n(X, \mathcal{K})$, the subalgebra of bounded functions by $C_n^*(X, \mathcal{K})$, and the subalgebra of functions vanishing at infinity by $C_{n0}(X, \mathcal{K})$. The symbol $\mathcal{K}$ will be omitted from these three expressions when it seems appropriate, and the subscript n will not be included when $n = 1$. Note that $L_n(C(X, \mathcal{K}))$ is a $\mathcal{K}$-algebra, and that there is a natural $\mathcal{K}$-isomorphism of $C_n(X, \mathcal{K})$ onto $L_n(C(X, \mathcal{K}))$. We shall use whichever $\mathcal{K}$-algebra is convenient at any point in the discussion. The same symbol will be used for a constant function and the matrix that is its value. For $F \in C_n(X)$, the ma-

Received December 9, 1966.

[1] The research of the second author was supported (in part) by a contract of the Office of Scientific Research, U. S. Air Force.