

# CORRECTION TO "CO-EQUALIZERS AND FUNCTORS"

BY  
K. A. HARDIE

G. M. Kelly has pointed out (Mathematical Reviews, vol. 35 (1968), #5491) that certain results in *Co-equalizers and functors* (this journal, vol. 11 (1967), pp. 336–348) are incorrect. There appear to be two distinct errors. The first is in the proof of the second assertion of Theorem 2.4: while the morphisms  $\beta GX$  are well defined,  $\beta$  is not a natural transformation. The second sentence in the statement of Theorem 2.4 should be deleted as well as the statement of Theorem 2.5. To compensate for the loss of 2.5 the second sentence of the statement of Theorem 2.7 should be altered to read: "If  $S$  is an  $X$ -functor in  $\mathbf{V}$ , if  $(\otimes, r)$  admits sections or if  $r$  is surjective and  $S$  is  $X$ -constructive then  $\alpha S$  is a co-equalizer of  $LR\alpha S$  and  $\alpha LRS$ ." No alteration to the proof of 2.7 is necessary. Corollary 2.10 can be left as it stands however it may be misleading since it is easy to show that if  $(\otimes, r)$  admits sections then every valuable  $X$ -germ is constructive. Theorem 0.2 must be altered to read: " $S$  is an  $X$ -functor in  $\mathbf{V}$  if and only if  $\alpha S$  is a co-equalizer of  $LR\alpha S$  and  $\alpha LRS$ ". The sentences in §3 relating to the proof of 0.2 should be deleted, the present form of 0.2 being a corollary of 2.7 as modified.

The second error is in the proof of Theorem 3.1, which is incorrect as stated. 3.1 should be altered to read:

THEOREM 3.1.  $\text{Ext}^n(C, -)$  is a  $(K_n \oplus P_n)$ -functor.

*Proof.* For every  $\Delta$ -module  $Y$  we certainly have

$$\text{Ext}^n(C, Y) \approx \text{Hom}(K_n, Y)/\chi^* \text{Hom}(P_n, Y)$$

where

$$0 \rightarrow K_n \xrightarrow{\chi} P_n \rightarrow P_{n-1} \rightarrow \cdots \rightarrow P_1 \rightarrow C \rightarrow 0$$

is an exact sequence with  $P_i$  projective ( $1 \leq i \leq n$ ). By 2.8,  $\text{Hom}(K_n, -)$  is a  $K_n$ -functor and  $\text{Hom}(P_n, -)$  is a  $P_n$ -functor. Hence  $\text{Hom}(K_n, -)$  and  $\text{Hom}(P_n, -)$  are both  $(K_n \oplus P_n)$ -functors. Thus the required result is a consequence of the following.

LEMMA. If  $S$  and  $T$  are  $X$ -functors, if  $Q \in \mathbf{V}$  and if  $w : T \rightarrow Q$  is a co-equalizer of  $u, v : S \rightarrow T$  then  $Q$  is an  $X$ -functor.

The lemma may be proved by an application of the  $3 \times 3$ -lemma for co-