HOMOMORPHISMS OF PRINCIPAL FIBRATIONS: APPLICATIONS TO CLASSIFICATION, INDUCED FIBRATIONS, AND THE EXTENSION PROBLEM

Dedicated to the memory of Tudor Ganea

BY
GERALD J. PORTER[1]

A fibration

$$F \xrightarrow{i} E \xrightarrow{p} B$$

is called principal if there exists an associative multiplication

$$\mu : F \times F \to F$$

and an associative action

$$\varphi : F \times E \to E$$

such that the following diagram commutes.

(I)
$$\begin{array}{ccccc} F \times F & \xrightarrow{1 \times i} & F \times E & \xrightarrow{* \times p} & B \\ \downarrow{\scriptstyle \mu} & & \downarrow{\scriptstyle \varphi} & & \| \\ F & \xrightarrow{i} & E & \xrightarrow{p} & B \end{array}$$

A fibre preserving map

$$g : (\mu, \varphi, F \to E \to B) \to (\mu', \varphi', F' \to E' \to B')$$

between principal fibrations is called a homomorphism if

(II)
$$\begin{array}{ccc} F \times E & \xrightarrow{(g|F) \times g} & F' \times E' \\ \downarrow{\scriptstyle \varphi} & & \downarrow{\scriptstyle \varphi'} \\ E & \xrightarrow{g} & E' \end{array}$$

commutes.

Principal fibrations and their homomorphisms are easily seen to form a category. The Dold-Lashof construction [2] is a functor from this category to the category of universal principal quasifibrations and their homomorphisms.

Homotopy commutativity of diagram (II) is not sufficient to ensure the existence of a map between the associated universal quasifibrations. However one is able to give higher homotopy conditions which, if satisfied, permit

Received May 23, 1969.
[1] The author was supported in part by a National Science Foundation grant.