

BLOCK EXTENSIONS

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Authors often tell us that their fictitious characters have wills of their own, and that they can grow and develop, during the writing of a long novel, in ways altogether unforeseen when the work was begun. The present article has some of the characteristics of these stubbornly independent literary creations. It started out as a simple observation—now quite buried in Theorem 10.20 below—that Brauer's First Main Theorem about blocks led to an isomorphism between certain group extensions associated with those blocks, an isomorphism which could be used as a reduction technique in the study of outer automorphisms of finite groups. During the initial write-up of this observation it developed that these group extensions behaved as if they were Clifford extensions $H[B]^*$ for blocks B of normal subgroups K of finite groups H , in the sense that the blocks of H lying over B could be computed from those of the twisted group algebra of $H[B]^*$. Furthermore, the original isomorphism became only a step in a reduction process paralleling Brauer's well-known analysis of blocks [1], a process yielding a reasonably simple formula for the Clifford extension $H[B]^*$ for the block B in terms of an ordinary Clifford extension for any of the conjugacy class of irreducible characters corresponding to B in Brauer's theory. Obviously one couldn't discuss either blocks or Brauer's analysis without a thorough study of defect groups, culminating in a method for computing the defect groups of a block of H lying over B from the defect groups of a corresponding block of the Clifford extension $H[B]^*$. Finally, the whole theory had to be put in suitable abstract settings (as in [3]) for the sake of possible generalizations as well as to clarify the actual content of the various theorems. Thus, from minor revisions to complete rewritings, from small improvements to whole new sections, the paper grew and expanded into a fullblown theory of block extensions in which the original observation is all but lost and any connection with outer automorphisms has completely disappeared.

Some of the maladjustments inherent in the manner in which this article grew are still visible in the final result, particularly in the choice of abstract settings. The axioms (2.1) used in the definition of the Clifford extension for a block and in the construction of the associated Clifford correspondence are quite suitable for the purpose, based as they are on the developed theory of [3] and [4]. However, when it came to defect groups and Brauer's analysis, no satisfactory fixed set of axioms was found. Indeed, throughout the part of the paper (§§4–9) devoted to these subjects the hypotheses change from section to section—sometimes even from theorem to theorem—in a most

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