PERTURBATION OF A SELF ADJOINT DIFFERENTIAL OPERATOR

BY

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Introduction

It is well known that if $f(x) \in L^p[0, \pi]$, $1 < p < \infty$, then, for example, its Fourier sine series converges to $f(x)$ in L^p. The Fourier sine series can, of course, be regarded as the eigenfunction expansion of $f(x)$ with respect to the self adjoint differential operator given by $B(u) = -(d/dx)^2(u)$ with boundary conditions $u(0) = u(\pi) = 0$.

Suppose now that B is a self adjoint differential operator given by $B(u) = (-1)^n(d/dx)^{2m}(u)$, for u in the domain of B. Let K denote a differential operator of the form,

$$p_{2m-2}(x)\left(\frac{d}{dx}\right)^{2m-2} + p_{2m-3}(x)\left(\frac{d}{dx}\right)^{2m-3} + \cdots + p_0(x)$$

where the $p_i(x)$ are bounded on $[0, \pi]$.

It is known (see [1] and [6]) that the eigenfunction expansion for $B + K$ converges in L^p, $1 < p < \infty$. We show, for $2 \leq p < \infty$, that this is implied by the corresponding statement for B. We establish this result by using the gaps in $\sigma(B)$, the spectrum of B, to obtain estimates for

$$\|R(\lambda; B + K) - R(\lambda; B)\|_{L^p \to L^p},$$

the norm of the difference of the resolvent operators, for λ on certain contours in the complex plane. In [7] D. R. Smart also uses the notion of gaps in the spectrum to obtain basically the same perturbation result, although the method of estimating the operator norms is different than ours.

1. Preliminaries

Let L denote the ordinary differential operator, $L = (-1)^m(d/dx)^{2m}$, $x \in I = [0, \pi]$. (We assume L is of even order merely for convenience.) Let U_i, $i = 1, 2, \ldots, 2m$, be independent boundary conditions. Thus we may write

$$U_i(f) = \sum_{j=0}^{2m-1} (a_{ij}f^{(j)}(0) + b_{ij}f^{(j)}(\pi)),$$

a_{ij}, b_{ij} being constants. We assume the boundary conditions are self adjoint and therefore also regular. (See [6].)

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