NONTRIVIAL LOWER BOUNDS FOR CLASS GROUPS OF INTEGRAL GROUP RINGS

BY

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To Olga Taussky, for her support and encouragement

1. Introduction

The aim of this paper is to give general lower bounds for the order of a subgroup $T(ZG)$ of the locally free class group Cl (ZG). $T(ZG)$ is generated by certain locally free modules (see Section 2) which have been considered [10], [15], [17] for their applications in algebraic topology. The lower bounds are expressed in terms of an important invariant of G, namely the Artin exponent $A(G)$. By definition $A(G)$ is the characteristic of the Grothendieck ring $G_0(QG)$ modulo the ideal generated by the image of the induction map $G_0(QC) \to G_0(QG)$, where C ranges over cyclic subgroups of G (see [16] for details). One knows [8], [9] that $A(G)$ divides the order of G and equals one iff G is cyclic. Our results assert:

THEOREM. *An odd prime p divides the order of the subgroup $T(ZG)$ of* Cl (ZG) *iff p divides the Artin exponent $A(G)$. Also* 2 *divides order of $T(ZG)$ if* 4 *divides $A(G)$ (assuming a Sylow 2-group of G is not dihedral).*

The formal properties of $T(ZG)$ imply it maps onto $T(ZG_0)$, G_0 obtained from G by quotient and subgroups. It follows that the proof reduces to certain groups that are among those considered in Section 3; for these we have a complete determination of $T(ZG)$. In the final analysis all the computations depend on special properties of units in group rings and orders. Our approach allows us to strengthen and extend several known results for noncyclic G and to show a common thread running through the arguments.

2. Definitions and formal properties

Let R be the ring of algebraic integers in an algebraic number field K, Λ an R-order in a finite-dimensional semisimple K-algebra A. Given an R-lattice X and prime p of K, X_p denotes completion at p (if p is infinite, set $R_p = K_p$ and $X_p = K_pX$). The class group Cl Λ is the Grothendieck group of the category of locally free left Λ-modules modulo the subgroup generated by free Λ-modules; (X) denotes the class in Cl Λ of a locally free module X. To get a sufficiently

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