SURGERY IN A FIBER BUNDLE

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Introduction

For an oriented smooth compact manifold M of dimension m let $\mathscr{L}_m(M)$ be the surgery space which is defined by Quinn in [9], [14, p. 240]. This space has the property that its homotopy groups are the Wall groups of $\pi_1(M)$. Let $\xi = (E, B, p, F)$ be an oriented smooth fiber bundle, i.e., E, B, F are oriented smooth compact manifolds, $p: E \to B$ is smooth and the orientation of E is induced by the orientations of B and F. Let $b = \dim B$ and $f = \dim F$. There is a pull back map $p^{\#}: \mathscr{L}_b(B) \to \mathscr{L}_{b+f}(E)$ [14, p. 240]. In this paper, we will study this map in the case that $\pi_1(B) = \pi_1(E) = Z_p$, a cyclic group of odd prime order p. At first, recall the following:

THEOREM 0.1 [14, p. 240]. *For a finitely generated group π, let $L_k(\pi)$ be the Wall group of π with trivial homomorphism $1: \pi \to Z_2$.*

(i) $\pi_i(\mathscr{L}_m(M)) = L_{m+i}(\pi_1(M))$.

(ii) $L_{4k}(1) = Z$ *and* $L_{4k+2}(1) = Z_2$.

(iii) *Let p be an odd prime, then* $L_{2k+1}(Z_p) = 0$, $L_{2k}(Z_p) = L_{2k}(1) \oplus \tilde{L}_{2k}(Z_p)$ *and* $\tilde{L}_{2k}(Z_p)$ *is a free abelian group of rank* $\frac{1}{2}(p-1)$.

Our main result is the following:

THEOREM 0.2. *Let ξ be a smooth fiber bundle as above with structure group H. If the identity component of H has a finite index, then for $i > 0$, $\pi_i(p^{\#})$ is given by*

$$\pi_i(p^{\#})x = \begin{cases} (I(F)x_1, I(F)x_2) & \text{if } f \equiv 0 \pmod 4 \text{ and } b + i \equiv 0 \pmod 4, \\ (\chi(F)x_1, I(F)x_2) & \text{if } f \equiv 0 \pmod 4 \text{ and } b + i \equiv 2 \pmod 4, \\ 0 & \text{otherwise,} \end{cases}$$

where $(x_1, x_2) = x \in L_{b+i}(1) \oplus \tilde{L}_{b+i}(Z_p)$, *and* $I(F)(\chi(F)$, *respectively*) *is the index* (*Euler characteristic, respectively*) *of* F.

This paper is organized as follows: In Section 1, we study the G-signatures of G-fibered manifolds. In Section 2, we apply the result of Section 1 and the theory of free G-bordism of Conner-Floyd to study the Atiyah-Singer invariants of free G-fibered manifolds. In Section 3, we will apply the result of Section 2 and the results of Sullivan to prove our main result.

1. The G-signatures of G-fibered manifolds

Let $p: E^{2n} \to B^{2m}$ be an oriented smooth fiber bundle with fiber F^{2k} where B, E may have nonempty boundaries. Then for each integer t, there is a bundle

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