SKEW PRODUCTS AND THE SIMILARITY OF VIRTUAL SUBGROUPS

BY

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Introduction

Consider a homomorphism $F: X \times H \to A$, where H and A are locally compact second countable groups and $X \times H$ is the virtual subgroup of H defined by applying Mackey's construction to a measure class preserving ergodic action of H on a finite analytic measure space X, m. We obtain (under some additional restrictions) an exact cohomology sequence (analogous to the first few terms of the Lyndon sequence for group extensions) involving $X \times H$, $W \times A =$ the range closure of F, and the measure groupoid $(X \times A) \times H$ defined by the skew product action (with respect to F) of H on $X \times A$, which plays the role of the kernel of F.

Then we are able to represent similar (in the sense of Ramsay) virtual subgroups of H and A, respectively, as contractions of a virtual subgroup of $H \times A$, obtained by the action of $H \times A$ on $X \times A$ that arises in Mackey's range closure construction. This representation leads easily to our main result—the similarity class of the Radon-Nikodym homomorphism for a virtual subgroup is a similarity invariant.

In Section 1 we introduce a method for dealing with inessential contractions (i.c.'s). Theorem 1.9 provides a method for dealing with homomorphisms as defined by Ramsay in [10], where the composition of homomorphisms is not necessarily defined, even though the composition of the similarity classes of the homomorphisms is defined. Also Theorem 1.9 leads easily to the first few terms of a Lyndon sequence for virtual groups in (2.4) in the special case where H and A are countable and the coefficient group B is an abelian analytic group. The part of the Lyndon sequence that is needed in greater generality is established in Theorems 2.5 and 2.6.

In Sections 3 and 4 we are concerned with similar virtual subgroups of H and A, respectively. 3.1 and 3.2 establish that consideration of a general half of a similarity $\beta: X \times H \to V \times A$ can always be reduced to consideration of the range closure homomorphism $\alpha: X \times H \to W \times A$ defined by a homomorphism $F: X \times H \to A$. Then the results of the previous sections on such an F are applicable. Theorem 3.3 and in more detail Lemma 3.4 provide the means for establishing the applications in Section 4.

The similarity invariance of the similarity class of the Radon-Nikodym homomorphism $[\rho]$ obtained in Theorem 4.4 allows us to classify (proper)

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