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REMARKS ON STRONGLY M-PROJECTIVE MODULES

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In [11], Varadarajan introduced the notion of strongly M-projective modules. He showed that every $B \in \operatorname{Mod} R$ satisfying $B\mathscr{A}(M) = 0$ possesses a strong M-projective cover if and only if $R/\mathscr{A}(M)$ is a right perfect ring where $\mathscr{A}(M)$ denotes the right annihilator of M in R. We show that if a certain class of modules in $\operatorname{Mod} R$ is closed under factors, then every $B \in \operatorname{Mod} R$ possesses a strong M-projective cover if and only if $R/\mathscr{A}(M)$ is right perfect, thereby conditionally extending Varadarajan's result to $\operatorname{Mod} R$. We also show via a pullback diagram that $B \in \operatorname{Mod} R$ is strongly M-projective if and only if $B/B\mathscr{A}(M)$ is a projective $R/\mathscr{A}(M)$-module. Varadarajan has shown this for the special case when $\mathscr{A}(M) = 0$.

If M is injective and $(\mathscr{T}, \mathscr{F})$ is the hereditary torsion theory on $\operatorname{Mod} R$ cogenerated by M, then it is shown that $B \in \operatorname{Mod} R$ is codivisible with respect to $(\mathscr{T}, \mathscr{F})$ if and only if B is strongly M-projective. From this it follows that if B has a projective cover, then B is codivisible with respect to $(\mathscr{T}, \mathscr{F})$ if and only if B is M-projective in the sense of G. Azumaya [1].

Throughout this paper R will denote an associative ring with identity and our attention will be confined to the category $\operatorname{Mod} R$ of unital right R-modules. We will often abuse notation and write $B \in \operatorname{Mod} R$ for an object of $\operatorname{Mod} R$. Furthermore all maps will be morphisms in $\operatorname{Mod} R$ while $\mathscr{A}(M)$ and M^J will denote the right annihilator of M in R and the direct product of the family $\{M_a = M\}$ $(a \in J)$ respectively. In addition, M will denote a fixed right R-module which is not necessarily injective.

Following Varadarajan [11], we call $B \in \operatorname{Mod} R$ strongly M-projective if every row exact diagram of the form

$$\begin{tikzcd} & B \arrow[dl, dashed] \arrow[d] & \\ M^J \arrow[r] & N \arrow[r] & 0 \end{tikzcd}$$

where J is any indexing set can be completed commutatively. This is a natural generalization of M-projective modules first studied by G. Azumaya [1]. Azumaya called B M-projective if the diagram above can be completed commutatively when J is a singleton.

If K is a submodule of $B \in \operatorname{Mod} R$, then K is said to be M-independent in B if for each $0 \neq x \in K$ there is an $f \in \operatorname{Hom}_R(B, M)$ such that $f(x) \neq 0$.

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