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SOME COHOMOLOGY INVARIANTS FOR DEFORMATIONS OF FOLIATIONS

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In this paper we examine some new cohomology invariants for deformations of foliations, or what we call n-foliations. An n-foliation of codimension q on M is a codimension $n + q$ foliation on $M \times I^n$ (where I^n is the unit n-cube $[0, 1]^n$) which intersects each slice $M \times \{x\}$, $x \in I^n$, as a codimension q foliation. Roughly speaking, these invariants are obtained by integrating the differential forms in the image of the map $WO_{q+n} \to \bigwedge^* (M \times I^n, \mathbf{R})$ over the fiber I^n. If certain conditions are satisfied, the resulting forms determine cohomology classes in $H^*(M, \mathbf{R})$. These classes have been examined in the case of 1-foliations in [8], and the construction given there uses Gelfand-Fuks cohomology. Their primary interest was the classes for 1-foliations which are the derivatives of deformable classes in $H^*(WOq)$. However, there are also many other classes for 1-foliations (and n-foliations) which cannot be interpreted in this way. A discussion of characteristic classes for deformations of foliations can also be found in [13, Section 8.7].

In Section 1 we give the constructions of these classes for C^∞ n-foliations and for complex holomorphic n-foliations. This construction also has a local form where, instead of integrating over the fiber I^n, one takes the interior product of a form from WO_{q+n} with the volume element $\partial/\partial t_1 \wedge \cdots \wedge \partial/\partial t_n$ on I^n. In this way one gets characteristic classes at each point $x \in I^n$ which contain local information about the deformation at that point.

Section 2 is concerned with the construction of non-trivial examples of these invariants. The basic idea is to look at cross products of n-foliations and m-foliations, obtaining $n + m$-foliations. The classes for the product then factor into products of classes for each of the factors. This fact was first pointed out in the case of (undeformed) foliations in [14]. However, it seems to be a much richer source of non-trivial examples for n-foliations than it is in the undeformed case. It is also worth noting that, in the examples constructed at the end of Section 2, the usual invariants for undeformed foliations all vanish on the individual foliations which make up the deformation.

If $B\Gamma q$ is the classifying space for codimension q Haefliger structures, a codimension q n-foliation on M determines a family of maps F_x: $M \to B\Gamma q$, parameterized by points $x \in I^n$. If the n-foliation has a non-trivial characteristic

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