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ALEXANDER MODULES OF SUBLINKS AND AN INVARIANT OF CLASSICAL LINK CONCORDANCE

BY

NOBUYUKI SATO

Introduction

A link in S^3 is an ordered collection $L = \{K_1, \ldots, K_m\}$ of smooth, oriented, pair-wise disjoint knotted circles in the 3-dimensional sphere. To any link, one may associate the *complement* $X = S^3 - \bigcup_{i=1}^{m} N(K_i)$, where $N(K_i)$ denotes an open tubular neighborhood of K_i in S^3. The *Alexander modules* of L are the homology groups of the universal abelian cover $\tilde{X}$ of X, viewed as modules over the integral group ring Λ of the group of covering transformations. By Alexander duality, this group is the free abelian group on m generators, so we may identify Λ with the Laurent polynomial ring

$$\mathbf{Z}[x_1, \ldots, x_m, x_1^{-1}, \ldots, x_m^{-1}].$$

We begin by studying the relation between the Alexander modules of a link

$$L = \{K_1, \ldots, K_m\}$$

in S^3 and those of a sublink $L' = \{K_2, \ldots, K_m\}$. The first result is the discovery of certain short exact sequences which express this relationship (see Theorem 1.1). An interesting corollary of this result is a new proof of a well-known formula of Torres [12] which related the Alexander polynomial of L to that of L', one which does not use the free differential calculus. The proof of the formula turns out to carry other important information as well. Investigation of a certain map (which is always zero in the cases of interest for the proof of the formula) leads to the discovery of a new link invariant $I_1(L)$ which detects non-boundary-linking. In many instances this invariant is quite easy to compute; in fact, in the examples in Section 3, it is far easier to compute than the Alexander polynomial. The proof of the fact that $I_1(L)$ vanishes for boundary links (Theorem 3.1) indicates that $I_1(L)$ is related to a certain rank invariant $r(L)$ which we define in Section 4. This invariant turns out to be an invariant of link concordance (Theorem 4.4). An application of $r(L)$ to the Whitehead link shows that it is not concordant to a boundary link, and therefore not a slice link. Finally, we show how $I_1(L)$ and $r(L)$ are related, and note that this implies that the examples of Section 3 are not concordant to boundary links.

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