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CHARACTERIZING 2-DIMENSIONAL MOISHEZON SPACES BY WEAKLY POSITIVE COHERENT ANALYTIC SHEAVES

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1. A considerable amount of effort has been expended in recent years on generalizing Kodaira's Embedding Theorem to a characterization of Moishezon spaces by some form of positive coherent sheaves. There are two possible approaches. One is to consider a relatively weak form of positivity; the problem is then to prove that a complex space carrying such a sheaf is necessarily Moishezon. The other is to give a stronger definition of positivity and then the problem is to show that every Moishezon space carries such a sheaf (see the bibliography in [8]).

One of the first and most natural positivity notions for sheaves was given by Grauert in his fundamental paper "Uber Modifikationen und exceptionelle analytische Mengen" ([4]). Let X be a reduced compact complex space and $\mathcal{S} \to X$ a coherent analytic sheaf. Grauert constructs a linear fibre space $V(\mathcal{S})$ dual to $\mathcal{S}$ such that $\mathcal{S}$ is the sheaf of linear forms on $V(\mathcal{S})$. He then calls $\mathcal{S}$ *weakly positive* if the zero-section of $V(\mathcal{S})$ is exceptional, that is, can be holomorphically contracted to a point. The main theorem of [4] is that a normal compact complex space is projective, and hence by Chow's theorem, projective algebraic, if and only if it carries a weakly positive locally-free sheaf. In light of this result, it seems quite natural to try to characterize Moishezon spaces by weakly positive coherent sheaves. Now it is a simple matter to prove that if X carries a weakly positive coherent sheaf then X is Moishezon (see [8]). The difficulty lies in showing that every Moishezon space carries such a sheaf.

In [8], I gave a slightly weaker definition of positivity than Grauert's. Let X, $\mathcal{S}$, and $V(\mathcal{S})$ be as above. The $V(\mathcal{S})$ is, in general, non-reduced and its reduction is, in general, not irreducible. Let V_R be the reduction of $V(\mathcal{S})$ and let π: $V_R \to X$ be the natural projection. Then there is an analytic set $A \subset X$ such that π: $V \mid (X - A) \to X - A$ is a vector bundle (see [10]). The primary component of $V(\mathcal{S})$, denoted V', is the closure in V_R of $V_R \mid X - A$. If X is irreducible, then V' is the unique irreducible component of V_R which is mapped onto X by π. Although V' is, in general, not a linear fibre space, it does have a well-defined zero-section and $\mathcal{S}$ is called *primary weakly positive* if

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