ILLINOIS JOURNAL OF MATHEMATICS
Volume 27, Number 4, Winter 1983

A RATIO ERGODIC THEOREM FOR GROUPS OF MEASURE-PRESERVING TRANSFORMATIONS

BY

MARIA E. BECKER

Introduction

In this paper we use the method introduced in ergodic theory by A. P. Calderón [1] combined with a covering lemma due to Besicovich to obtain the pointwise convergence of averages formed with n-parameter groups of measure-preserving transformations.

Let $(X, \mathscr{A}, \mu)$ be a σ-finite measure space. By an n-parameter group of measure-preserving transformations we mean a system of mappings $(\theta_t, t \in R^n)$ of X into itself having the following properties:

(i) $\theta_t(\theta_s x) = \theta_{t+s}x$; $\theta_0 x = x$ for every t and s in R^n and every x in X.

(ii) for every measurable subset E of X, $\theta_t(E)$ is measurable and its measure equals the measure of E, for any t in R^n.

(iii) For any function f measurable on X, the function $f(\theta_t x)$ is measurable on the product space $R^n \times X$, where the euclidean space R^n is endowed with Lebesgue measure.

Let p be a non-negative function in $L^1(\mu)$. For each function f integrable over X, we consider the ratios

$$R_\alpha(f, p)(x) = \frac{\int_{B_\alpha} f(\theta_t x)dt}{\int_{B_\alpha} p(\theta_t x)dt} \quad \text{if} \int_{B_\alpha} p(\theta_t x)dt > 0,$$

$R_\alpha(f, p)(x) = 0$ otherwise, where B_α is the ball in R^n of radius α and center at the origin.

In what follows we give sufficient conditions for the almost everywhere convergence of $R_\alpha(f, p)$, as $\alpha \to \infty$, in the set where the denominators eventually become positive and therefore it arises a continuous version of the Chacon and Ornstein theorem [2].

If f is integrable over X, we denote $\int_X f\, d\mu$ by $\int f(x)dx$.

Received July 23, 1981.