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CENTRAL LIMIT THEOREMS IN A FINITELY ADDITIVE SETTING[1]

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1. Introduction

Let I be an arbitrary nonempty set. Let I^* denote the set of all finite sequences of elements of I including the empty sequence and $P(I)$ denote the set of all finitely additive probabilities defined on all subsets of I. A *strategy* σ on I is a mapping on I^* into $P(I)$. The strategy σ is called an *independent strategy* if there exists a sequence $\{\gamma_n\}_{n\geq 1}$ of elements of $\mathrm{P}(I)$ such that $\sigma(p) = \gamma_{n+1}$ whenever p is an element of I^* of length n, $n \geq 0$. (The empty sequence has length zero.) In this case we shall denote σ by

$$\gamma_1 \times \gamma_2 \times \ldots \times \gamma_n \times \ldots .$$

If $\gamma_n = \gamma_m$ for all n, $m \geq 1$, then σ will be called an *i.i.d strategy*. If

$$\sigma(i_1, \ldots, i_n) = \sigma(i_n)$$

for all $n \geq 1$ and all $i_1, \ldots, i_n \in I$, then σ will be called a *Markov strategy with stationary transitions*. Let N stand for the set of positive integers and equip $H = I^N$ with the product of discrete topologies. Let $\mathscr{B}$ be the σ-field of subsets of H generated by open sets. Following Dubins and Savage ([4] and [3]), and Purves and Sudderth [9], it can be shown that every strategy σ induces a finitely additive probability on $\mathscr{B}$, unique subject to certain regularity conditions. We shall conveniently denote this probability on $\mathscr{B}$ by σ again. To state the central limit theorems in a finitely additive setting, we shall call a sequence $\{Y_n\}_{n\geq 1}$ of real valued functions on H *a sequence of coordinate mappings* if $Y_n(h)$ depends only on the n-th coordinate of h for all $h \in H$.

THEOREM 1A (Lindeberg theorem). *Let σ be an independent strategy on I. Let $\{Y_n\}_{n\geq 1}$ be a sequence of coordinate mappings on H such that*

$$\int Y_n(h)d\sigma(h) = 0 \quad \textit{and} \quad 0 < \int Y_n^2(h)d\sigma(h) < \infty$$

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[1]This work was done when the author was at the Indian Statistical Institute, Calcutta.

[2]After the author proved these theorems, R.L. Karandikar obtained a more general result in [7]. We still indicate proofs of these theorems, firstly because several of the techniques and lemmas used in the proof are needed in Section 3. Moreover our proofs are more direct and elementary.

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