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BIVARIATE CARDINAL INTERPOLATION BY SPLINES ON A THREE-DIRECTION MESH

BY

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**Dedicated to I.J. Schoenberg
to whose insight and sense of beauty we are all indebted**

1. Introduction

In this paper, we carry Schoenberg's beautiful cardinal spline theory [S_2], [S_3] over to a two-dimensional context which is not just the tensor product of the univariate situation. We find that we must work harder, yet must be satisfied with less precise results.

We are after a bounded cardinal interpolant to bounded data. This means that we are looking for a function of the form

$$If = \sum_{j\in\mathbf{Z}^2} a_j M(\cdot - j)$$

with $a \in l_\infty(\mathbf{Z}^2)$ which agrees with a given bounded function f on $\mathbf{Z}^2$. Here, M is a fixed function of compact support. In Section 2, we follow Schoenberg [S_1] in describing necessary and sufficient conditions on the Fourier transform of M to insure the correctness of the interpolation problem, i.e., the existence and uniqueness of solutions.

We are particularly interested in using for M a box spline, i.e., the two-dimensional "shadow" of an m-dimensional cube, as given explicitly in (1) below. Let Z be a set of vectors in $\mathbf{R}^2$. We find it convenient to change the definition [BH_1]

$$M\phi := \int_{[0,1]^Z} \phi\Big(\sum_{\zeta\in Z} \lambda(\zeta)\zeta\Big)\, d\lambda$$

of the box spline $M = M_Z$ to include an appropriate shift which makes the

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