

A COMPLETE CHARACTERIZATION OF STOCHASTIC PROCESSES DEFINED ON REGULAR SEMIGROUPS WITH APPLICATIONS TO STOCHASTIC MODELS

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1. Introduction

It is the purpose of our paper to show that the whole is greater than the sum of its parts. Numerous papers have been written concerning various properties of probability functions defined on semigroups. The simplest structures of discrete and compact semigroups were considered first (see Martin-Lof [7] and Mukherjea and Tserpes [11]). Then the completely simple case was dealt with for properties of random walks (see [11] and [1]). The completely regular case has been shown by the author in [2]. Also, the special case of matrix semigroups has been studied relying primarily on the properties of rank (see Hognas and Mukherjea [5]). We intend to show that these separate studies can be combined into one harmonious concept. By doing so, the properties become far more flexible and can be used to prove some unsolved conjectures.

In order to do so, we first need to define the necessary terminology. Most of this information is available in Clifford and Preston [3], Paalman-deMiranda [12], Petrich [13], and Mukherjea and Tserpes [11]. Let S be a locally compact, Hausdorff, second countable topological semigroup. An element $x \in S$ is regular provided that there exists some $y \in S$ such that $x = xyx$. S is completely regular if $x = xyx$ and $xy = yx$; S is a union of maximal pairwise disjoint subgroups. The properties of S are listed in Clifford and Preston [3].

S is a completely simple semigroup if it contains no proper ideals and contains an idempotent minimal with respect to the partial ordering $e \leq f$ if $ef = fe = e$. If S is completely simple then we can write $S = X \times G \times Y$ where $G = eS \cap Se = eSe$, $X = E(Se)$, and $Y = E(eS)$ where e is a minimal idempotent and the notation $E(A)$ denotes the set

$$\{f \in A: f \text{ is idempotent}\}.$$

Note that $S = \bigcup_{g_x, g_y} \{g_x\} \times G \times \{g_y\}$ is a union of maximal disjoint semigroups and hence it is also completely regular.

We intend to consider several probabilistic concepts in this paper. Therefore it is also necessary to define these terms for semigroups. Let X have law μ . We

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