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A GENERALIZATION OF FRÖHLICH'S THEOREM TO WILDLY RAMIFIED QUATERNION EXTENSIONS OF Q

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1. Introduction

Let N/K be a finite normal extension of number fields and let $G = \mathrm{Gal}(N/K)$. By E. Noether's theorem (cf. [5, p. 26–27]), the ring of integers o_N of N is projective as a G-module if and only if N/K is at most tamely ramified. In [14], M. Taylor proved that in this case, $(o_N) - [K\colon \mathbf{Q}](\mathbf{Z}[G]) = W_{N/K}$ where (o_N) is the class of o_N in $K_0(\mathbf{Z}[G])$ and $W_{N/K}$ is the Cassou-Noguès Fröhlich class of N/K (cf. [2, p. 18–19], [5]). The group $K_0(\mathbf{Z}[G])$ is the Grothendieck group of all finitely generated G-modules of finite projective dimension and the class $W_{N/K}$ is defined by means of the Artin root numbers of the irreducible symplectic representations of G.

Let rank: $K_0(\mathbf{Z}[G]) \to \mathbf{Z}$ be the homomorphism by

$$\mathrm{rank}((A)) = \mathrm{rank}_{\mathbf{Q}[G]} \mathbf{Q} \otimes_{\mathbf{Z}} A$$

if A is finitely generated and of finite projective dimension. The class group $\mathrm{Cl}(\mathbf{Z}[G])$ of G is defined to be the kernel of rank. In [3], T. Chinburg defined Galois invariants $\Omega(N/K, i)$ of N/K in $\mathrm{Cl}(\mathbf{Z}[G])$ and proved that $\Omega(N/K, 2) = (o_N) - [K\colon \mathbf{Q}](\mathbf{Z}[G])$ for all N/K which are at most tamely ramified.

Since both classes, $\Omega(N/K, 2)$ and $W_{N/K}$, are defined for all N/K, and not only for those which are tamely ramified, one may ask the following question.

QUESTION (Chinburg [3]). Is $\Omega(N/K, 2) = W_{N/K}$ for all N/K?

Here we will prove the following result.

THEOREM 1. *Suppose that $K = \mathbf{Q}$ and that G is isomorphic to the quaternion group H_8 of order eight. If there are at least two places over the prime 2 in N then $\Omega(N/\mathbf{Q}, 2) = W_{N/\mathbf{Q}}$.*

The techniques of this paper apply as well to the case in which there is exactly one place over the prime 2 in N. We believe that further computation will determine whether the conclusion of the theorem holds in this case.

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