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THE NUMBER OF ELEMENTS REQUIRED TO DETERMINE $(p, 1)$-SUMMING NORMS

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Introduction

A basic problem concerning the various summing norms of operators is to estimate the number of elements needed to determine the value of the norm (up to a constant multiple), either in terms of the rank of the operator or in terms of the dimension of its domain. As well as being of interest in their own right, such estimates play a vital part in evaluating ratios between different summing norms.

A good summary appears in [10], Chapter 4. The most satisfactory results concern the norm π_2: if the rank of T is n, then n elements are enough to give $(1/\sqrt{2})\pi_2(T)$ (this, the prototype of all theorems of this type, was originally proved in [9], with 2 instead of $\sqrt{2}$), and the exact value can be found using $\frac{1}{2}n(n+1)$ elements in the real case, or n^2 elements in the complex case. For other p, one finds that 4^n elements will give at least $\frac{1}{3}\pi_p(T)$, and when $p = 1$, the number of elements needed is of this order. More recently, Szarek [8] has shown that for operators on an n-dimensional space, π_1 can be estimated using $n \log n$ elements, and this result has been extended to other p by Johnson and Schechtman [6].

For the mixed summing norms $\pi_{p,2}$, König [7] showed that there is a constant C such that

$$\pi_{p,2}(T) \leq Cp/(p-2)\pi_{p,2}^{(n)}(T)$$

for operators of rank n. In [8] it was shown that $Cp/(p-2)$ can be replaced by a constant independent of p, and in [2] the constant was improved to $\sqrt{2}$. By applying some fairly deep theorems, these results can be applied to derive corresponding ones for $\pi_{p,1}$, where $p > 2$ [10, Proposition 24.9], but with intervening constants C_p that tend to infinity as $p \to 2$.

In this note we show by the easiest of arguments (simply discarding some of the elements) that the number of elements required for $\pi_{p,1}$ is not more than $c_p(\alpha/\beta)^{p^*}$, where $\alpha = \pi_1(T)$, $\beta = \pi_{p,1}(T)$ and c_p grows large when $p \to 1$. This converts the problem into estimating α/β. For the case $p = 2$, a result from [4] shows that α/β is of the order of $(n \log n)^{1/2}$ for operators of

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