ILLINOIS JOURNAL OF MATHEMATICS
Volume 39, Number 2, Summer 1995

A NOTE ON CONFORMAL VECTOR FIELDS AND POSITIVE CURVATURE

DaGang Yang[1]

Introduction

All known examples of compact Riemannian manifolds with positive sectional curvature carries a positively curved metric with a continuous Lie group as its group of isometries and thus carries a nontrivial vector field of infinitesimal isometries, i.e., a Killing vector field. However, due to a theorem of M. Berger [1], such a Killing vector field must be singular at least at one point if the manifold is even dimensional. This is related to a well-known conjecture that for an even dimensional closed positively curved Riemannian manifold, its Euler characteristic is positive (cf. [4]). It is easy to see that Berger's theorem remains true for conformal vector fields (see also [3]). On the other hand, the Euler characteristic of a closed odd dimensional manifold is always zero. There are many simple examples of odd dimensional closed positively curved Riemannian manifolds which carry nonsingular Killing vector fields. The simplest example is perhaps the round 3-dimensional sphere S^3, which admits 3 pointwise linearly independent Killing vector fields while any two of them do not commute. This is obvious if one considers S^3 from the Lie group theoretic point of view.

The aim of this note is to give a generalization of M. Berger's theorem to odd dimensional manifolds.

THEOREM. *On a closed odd dimensional Riemannian manifold of positive sectional curvature, each pair of commutative conformal vector fields are dependent at least at one point.*

Remark 1. Analogous to the above mentioned conjecture, one might expect the following: On a closed positively curved odd dimensional Riemannian manifold, each pair of commutative vector fields are dependent at least at one point.

Received December 1, 1992.
1991 Mathematics Subject Classification. 53C.
[1]Partially supported by a grant from the National Science Foundation.