TRACE IDEAL CRITERIA FOR OPERATORS OF HANKEL TYPE

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In this paper we obtain trace ideal criteria for commutators of multiplication operators with integral operators having kernels of a critical homogeneity. The prototype for the class of operators considered is the Hankel operator associated with the Bergman projection. Specifically, let D be a domain in \mathbb{C}^n , and let P be the orthogonal projection of $L^2(D)$ onto its holomorphic subspace $A^2(D)$. When f is a function on D, the Hankel operator with symbol f is defined formally by

$$H_f = (I - P)M_f P = [M_f, P]P,$$

where $[M_f, P]$ is the *commutator* defined by

$$[M_f, P] = M_f P - P M_f$$

and M_f is the multiplication operator defined by $M_f g = fg$.

Trace ideal criteria for Hankel operators with conjugate holomorphic symbols have been obtained in various settings by several authors [2], [8], [12], [14], [19]. For general symbols, trace ideal criteria for the commutators $[M_f, P]$, and hence also for H_f , were first obtained in the unit ball by K. Zhu [21], and later in bounded symmetric domains by D. Zheng [20]. More recently, D. Luecking [15] has given a direct characterization of the symbol classes for trace ideals of Hankel operators in the unit disk, without the mediation of the commutator operators that was used in earlier work. (Of course, the distinction between H_f and $[M_f, P]$ evaporates when the symbol f is conjugate holomorphic, as in the case of classical Hankel operators.)

The purpose of this paper is to extend the results of [21] and [20] to a general class of operators which are loosely modeled on the operators H_f . As special cases, we will obtain trace ideal criteria for Hankel operators on strictly pseudoconvex domains in \mathbb{C}^n and on finite type domains in \mathbb{C}^2 (see

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