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UNIFORM EXTENDIBILITY OF THE BERGMAN KERNEL

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1. Introduction

In this note we simplify the proof of and extend a theorem of S-C Chen on the uniform extendibility of the Bergman kernel function and its relation to global regularity properties of the Bergman projection. Applications are given to Bergman kernel density and finite order vanishing theorems which arise in mapping problems between equidimensional domains.

Suppose Ω is a bounded domain in $\mathbf{C}^n$. The Bergman projection P associated to Ω is the orthogonal projection of $L^2(\Omega)$ onto $H^2(\Omega)$ where $H^2(\Omega)$ denotes the closed subspace of $L^2(\Omega)$ consisting of square integrable holomorphic functions on Ω. The Bergman kernel function $K(z,w)$ is the kernel function for P and satisfies

$$Pu(z) = \int_\Omega K(z,w)u(w)\,dV_w \quad \text{for all } u \in L^2(\Omega).$$

We say that condition Q holds on Ω if P maps $C_0^\infty(\Omega)$ into $\mathscr{O}(\overline{\Omega})$, where $\mathscr{O}(\overline{\Omega})$ denotes the space of holomorphic functions on Ω that can be extended holomorphically to some open set containing $\overline{\Omega}$. Finally, we let $\mathscr{O}(\Omega)$ denote the space of all holomorphic functions on Ω and $A^\infty(\Omega)$ denote the space of holomorphic functions on Ω that are in $C^\infty(\overline{\Omega})$.

Extendibility properties of the Bergman kernel are important in the study of extension of biholomorphic and proper holomorphic mappings between domains in $\mathbf{C}^n$. It has been shown that extendibility of the mappings can be deduced from extendibility properties of the Bergman kernel, which follow from global regularity properties of the Bergman projection (see Bell [1]).

It is known that condition Q holds on a domain Ω whenever the $\bar{\partial}$-Neumann problem is globally real analytic hypoelliptic on Ω; for instance see Bell [2]. This property of the $\bar{\partial}$-problem is known to hold in strictly pseudo-

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