

BROWNIAN MOTIONS ON A HALF LINE

Dedicated to W. Feller

BY

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Contents

1. The classical Brownian motions.
2. Feller's Brownian motions.
3. Outline.
4. Standard Brownian motion: stopping times and local times.
5. Brownian motion on $[0, +\infty)$.
6. Special case: $p_+(0) < 1$.
7. Green operators and generators: $p_+(0) = 1$.
8. Generator and Green operators computed: $p_+(0) = 1$.
9. Special case: $p_2 = 0 < p_3$ and $p_4 < +\infty$.
10. Special case: $p_2 > 0 = p_4$.
11. Increasing differential processes.
12. Sample paths: $p_1 = p_3 = 0 < p_4$ ($p_2 > 0/p_4 = +\infty$).
13. Simple Markovian character: $p_1 = p_3 = 0$ ($p_2 > 0/p_4 = +\infty$).
14. Local times: $p_1 = p_3 = 0$ ($p_2 > 0/p_4 = +\infty$).
15. Sample paths and Green operators:

$$p_1 u(0) + p_3 (\mathcal{G}u)(0) = p_2 u^+(0) + \int_{0+} [u(l) - u(0)] p_4(dl)$$

$$(p_2 > 0/p_4 = +\infty).$$

16. Bounded interval: $[-1, +1]$.
17. Two-sided barriers.
18. Simple Brownian motions.
19. Feller's differential operators.
20. Birth and death processes.

Numbering. 1 means formula 1 of the present section; 2.1 means formula 1 of Section 2, etc.; the numbering of the diagrams is similar.

1. The classical Brownian motions

Consider the space of all (continuous) *sample paths* $w: [0, +\infty) \rightarrow R^1$

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