# FLAG-TRANSITIVE COLLINEATION GROUPS OF FINITE PROJECTIVE SPACES

BY

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## 1. Introduction

A *flag* in a projective space $\mathcal{P}$ of dimension $d \geqq 2$ is a sequence

$$S_0 \subset S_1 \subset \cdots \subset S_{d-1}$$

of linear subvarieties of $\mathcal{P}$ such that $S_i$ has dimension $i$ $(i = 0, 1, \cdots, d-1)$. Thus, for example, a flag in a projective plane is an incident point-line pair. A collineation group $G$ of $\mathcal{P}$ will be called *flag-transitive* if any one flag can be carried onto any other by some collineation in $G$. The *little projective group* of a Desarguesian $\mathcal{P}$ (i.e., the group generated by all elations of $\mathcal{P}$, isomorphic with $PSL_{d+1}(F)$, where $F$ is the coordinatizing field) is flag-transitive. Thus the following theorem can be considered as giving a geometric characterization of $PSL_{d+1}(F)$ for finite $F$. If the number of points on each line of $\mathcal{P}$ is $n + 1$ we will refer to $n$ as the *order* of $\mathcal{P}$. (For $d > 2$ this differs from the order of the symmetric design formed by the points and hyperplanes of $\mathcal{P}$.)

THEOREM. *A flag-transitive collineation group $G$ of a Desarguesian projective space $\mathcal{P}$ of dimension $d \geqq 2$ and finite order $n$ must contain the little projective group of $\mathcal{P}$ unless*

(a) $d = 2$, $n = 2$, *and* $|G| = 3 \cdot 7$, *or*

(b) $d = 2$, $n = 8$, *and* $|G| = 9 \cdot 73$, *or*

(c) $d = 3$, $n = 2$, *and $G$ is isomorphic with the alternating group $A_7$ of degree* 7.

For $d = 2$ this theorem coincides with Theorem 1 of [6]. The extension to dimensions $\geqq 3$ (where, of course, the Desarguesian property necessarily holds) is obtained in this paper as an application of extensions of results of André [1], Gleason [5], and Wagner [9] concerning perspectivities, together with a special result about embeddings of $PSL_k(F)$ in $PGL_{k+l}(F)$.

The exceptions stated in the theorem are real. In case (c), $G \approx A_7$ is doubly transitive on the points of $\mathcal{P}$. Concerning the question whether the Desarguesian condition can be moved from the hypotheses to the conclusion of the theorem, i.e., whether the existence of a flag-transitive collineation group on a finite projective plane implies Desargues' Theorem, see [6].

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