ON UNIVERSAL TRANSFORMATION GROUPS

BY

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I. Introduction

In this paper, we characterize minimal sets (X, T, π) where X is Tychonoff (see [6]) by algebras of continuous functions, study compactifications of a transformation group, and prove that there is a unique universal compactification up to isomorphism of transformation groups. We develop several algebraic-topology and Banach-algebra properties for the universal minimal set associated with a discrete group (see [3]). We define a universal almost periodic minimal set associated with any topological group and prove there is a unique universal almost periodic minimal set associated with a topological group up to homeomorphism of spaces. In particular, we show that the phase space of an almost periodic minimal set (X, T, π) with compact Hausdorff space X is homeomorphic to a quotient space of a topological group $L(T)$, which is the maximal ideal space of the algebra of all left almost periodic functions on T. In the last section, we define a universal minimal set associated with any topological group and prove there is a unique universal minimal set up to isomorphism, which is a generalization of a result of Professor R. Ellis (see [3]). As a general reference for the notions occurring here consult [6] and [9]. The author wishes to take this opportunity to express his indebtedness to Professor W. H. Gottschalk and Professor H. C. Wang for their encouragement and direction.

II. The general case

Let (X, T, π) be a transformation group with Tychonoff phase space X. Let $C^*(X, R)$ and $C^*(T, R)$ be the algebras of all bounded, continuous, real-valued functions on X and on T, respectively, with the uniform norm. For each $t \in T$, we define

$$(\pi^*)^t : C^*(X, R) \to C^*(X, R) \quad \text{by} \quad (x)(f(\pi^*)^t) = (x\pi^t)f$$

for $f \in C^*(X, R)$ and $x \in X$, and

$$(\rho^*)^t : C^*(T, R) \to C^*(T, R) \quad \text{by} \quad (s)(g(\rho^*)^t) = (st)g$$

for $g \in C^*(T, R)$ and $s \in T$, respectively. Then t is an algebra-isomorphism. Let T_d be the set of all these t, for $t \in T$, with the discrete topology. Then T_d is an automorphism group of $C^*(X, R)$ and $C^*(T, R)$, respectively. Thus, we have

LEMMA 1. (1) *These* $(C^*(X, R), T_d, \pi^*)$ *and* $(C^*(T, R), T_d, \rho^*)$ *are transformation groups.*

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