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RESIDUES AND DIFFERENTIAL OPERATORS ON SCHEMES

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CONTENTS

0. Introduction. Suppose X is a finite-type scheme over a field k, with structural morphism π. Consider the twisted inverse image functor $\pi^!: \mathsf{D}^+_{\mathrm{c}}(k) \to \mathsf{D}^+_{\mathrm{c}}(X)$ of Grothendieck duality theory (see [Ha1]). The *residue complex* $\mathcal{K}^{\cdot}_X$ is defined to be the Cousin complex of $\pi^! k$. It is a bounded complex of quasi-coherent $\mathcal{O}_X$-modules, possessing remarkable functorial properties. In this paper we provide an explicit construction of $\mathcal{K}^{\cdot}_X$. This construction reveals some new properties of $\mathcal{K}^{\cdot}_X$ and also has applications in other areas of algebraic geometry.

Grothendieck duality, as developed by Hartshorne in [Ha1], is an abstract theory, stated in the language of derived categories. Even though this abstraction is suitable for many important applications, one often wants more explicit information. Thus, a significant amount of work was directed at finding a presentation of duality in terms of differential forms and residues. Mostly, the focus was on the dualizing sheaf ω_X, in various circumstances. The structure of ω_X as a coherent $\mathcal{O}_X$-module and its variance properties are thoroughly understood by now, thanks to an extended effort including [Kl], [KW], [Li], [HK1], [HK2], [LS], and [HS]. Regarding an explicit presentation of the full duality theory of dualizing complexes, there have been some advances in recent years, notably in the papers [Ye1], [SY], [Hu], [Hg], and [Sa].

In this paper we give a totally new construction of the residue complex $\mathcal{K}^{\cdot}_X$, when k is a perfect field of any characteristic and X is any finite-type k-scheme. The main idea is the use of *Beilinson completion algebras* (BCAs), introduced in

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