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ON CR MAPPINGS BETWEEN PSEUDOCONVEX HYPERSURFACES OF FINITE TYPE IN $\mathbb{C}^2$

BERNARD COUPET AND ALEXANDRE SUKHOV

0. Introduction. This paper is devoted to the study of regularity of Cauchy-Riemann mappings between smooth pseudoconvex hypersurfaces of finite type in $\mathbb{C}^2$. Our main result is the following.

THEOREM 0.1. *Let $f: \Gamma_1 \to \Gamma_2$ be a nonconstant Hölder $1/2$-continuous* CR *mapping between C^∞ smooth pseudoconvex hypersurfaces of finite type in $\mathbb{C}^2$. Then f is a locally finite-to-one mapping of class C^∞, and for every point p in Γ_1 the type of Γ_2 at the point $f(p)$ divides the type of Γ_1 at p. (If Γ_j are real analytic, then f is real analytic as well.)*

We emphasize that this is a purely local assertion, since Γ_j are not supposed to be compact.

We describe now the history of the question. The intensive study of the boundary behaviour of holomorphic mappings in several complex variables began after the fundamental paper of Fefferman [26], where he proved that a biholomorphism between (C^∞) strictly pseudoconvex domains extends smoothly up to the boundary. Together with an outstanding work of Chern-Moser [14], this result showed the importance of the boundary properties of holomorphic mappings for general problems of geometric complex analysis on domains in $\mathbb{C}^n$.

Fefferman's original proof was based on deep analysis of the boundary behaviour of the Bergman kernel and geometry of geodesics of the Bergman metric of a strictly pseudoconvex domain. Later, several new approaches were introduced that allowed extension of the Fefferman mapping theorem to a large class of pseudoconvex domains.

Lempert gave a self-contained geometric proof of Fefferman's theorem based on his theory of boundary behaviour of extremal discs for the Kobayashi metric [34]. Another approach was proposed by Nirenberg-Webster-Yang [37]. They deduced the Fefferman theorem from a nonanalytic version of the Lewy-Pinchuk reflection principle [35] and [41]. Their techniques were simplified and pushed further by Pinchuk-Khasanov [42], Fornaess-Low [27], and Coupet [17].

A quite different direction was discovered by Bell and Ligocka (see, for instance, [10]); they observed that the Fefferman mapping theorem follows from regularity properties of the Bergman projector. This approach relates Fefferman's mapping theorem to deep results on the $\bar{\partial}$-analysis on pseudoconvex domains. (For instance, the desired regularity of the Bergman projection follows from the

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