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KK-THEORY OF REDUCED FREE-PRODUCT C*-ALGEBRAS

EMMANUEL GERMAIN

0. Introduction. For unital C*-algebras endowed with states, there is a natural, reduced, free-product construction that generalizes the C*-algebra of the regular representation of a free-product group. Whereas the question of computing the K-theory and all the various KK-groups is completely understood in the case of discrete groups (see the work of Pimsner in [13]), little is known so far for more general situations.

In this paper, we prove an analogue of Cuntz's K-amenability result (see [3]), showing that there is a K-theoretical equivalence between the reduced and full free product of nuclear C*-algebras. This shows in particular that the reduced free products obtained for different choices of states (with a natural restriction) are all K-equivalent.

Since it is often easier to compute the K-theory of the full free product of C*-algebras, our result is an important step on our way to understand the reduced free product C*-algebra at a K-theoretical level.

Furthermore, the tools developed here allow us to give a unified treatment and to extend to a larger set of C*-algebras the computation of the KK-groups of full free-product C*-algebras. To this purpose, we introduce the notion of K-pointed C*-algebras. This is the K-theoretic generalization of a unital C*-algebra endowed with a one-dimensional representation. For those algebras, we prove the exact sequences conjectured by Cuntz in [4] by an argument that generalizes his proof for C*-algebras endowed with a one-dimensional representation.

This work is organized as follows.

Sections 1 and 2 consist of preliminaries needed for the constructions of Sections 3 and 4. The first section contains a few lemmas about the homogeneous space of finite sets of orthonormal vectors in the canonical stable A-Hilbert module. As for Hilbert spaces, we show that unitaries which are compact perturbations of the identity act transitively on these sets.

In Section 2, we recall technical nuclearity results for Kasparov bimodules. As shown in [16], for any faithful representation of a separable C*-algebra A, there is a presentation of the identity Kasparov bimodule for A by a bimodule with this representation as left action. Moreover, if A is nuclear, we can also construct a homotopy between this module and the canonical identity bimodule which in some sense "lies" in the given representation.

Let A_r denote now the reduced free-product C*-algebra of a set of K-nuclear

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