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EQUIVARIANT INDEX FORMULAS FOR ORBIFOLDS

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1. Introduction. Let P be a smooth manifold. Let H be a compact Lie group acting on P. We assume that the action of H is infinitesimally free, that is, the stabilizer $H(y)$ of any point $y \in P$ is a finite subgroup of H. We write the action of H on the right. The quotient space P/H is an orbifold. (If H acts freely, then P/H is a manifold.) Reciprocally, any orbifold M can be presented this way: for example, one might choose P to be the bundle of orthonormal frames for a choice of a metric on M and $H = O(n)$ if $n = \dim M$. We will assume that there is a compact Lie group G acting on P such that its action commutes with the action of H. We will write the action of G on the left. Then the space P/H is provided with a G-action. Such data (P, H, G) will be our definition of a presented G-orbifold. We will say shortly that P/H is a G-orbifold.

Consider a compact G-orbifold P/H. A tangent vector on P tangent at $y \in P$ to the orbit $H \cdot y$ will be called a vertical tangent vector. Let T_H^*P be the subbundle of T^*P orthogonal to all vertical vectors. We will say that T_H^*P is the horizontal cotangent space. We denote by (y, ξ) a point in T^*P. Consider two $(G \times H)$-equivariant vector bundles $\mathscr{E}^\pm$ on P. Let $\Gamma(P, \mathscr{E}^\pm)$ be the spaces of smooth sections of $\mathscr{E}^\pm$. Let

$$\Delta \colon \Gamma(P, \mathscr{E}^+) \to \Gamma(P, \mathscr{E}^-)$$

be a $(G \times H)$-invariant differential operator. Consider the principal symbol $\sigma(\Delta)$ of Δ. The operator Δ is said to be H-transversally elliptic if

$$\sigma(\Delta)(y, \xi_0) \colon \mathscr{E}_y^+ \to \mathscr{E}_y^-$$

is invertible for all $\xi_0 \in (T_H^*P)_y - \{0\}$. When Δ is H-transversally elliptic, the equivariant index of Δ is defined as in [1] and is a trace-class virtual representation of $G \times H$. Introduce $(G \times H)$-invariant metrics on P and on $\mathscr{E}^\pm$. Let Δ^* be the formal adjoint of Δ. The virtual space $Q(\Delta)$ of H-invariant "solutions" of Δ

$$Q(\Delta) = [(\mathrm{Ker}(\Delta))^H] - [(\mathrm{Ker}(\Delta^*))^H]$$

is a finite-dimensional virtual representation space for G. More generally, we consider $(G \times H)$-transversally elliptic operators on P. Then the space $Q(\Delta)$ of H-invariant "solutions" of Δ is a trace-class virtual representation of G.

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