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# ON THE AFFINE ANALOGUE OF JACK AND MACDONALD POLYNOMIALS

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**Introduction.** Jack and Macdonald polynomials are an important class of symmetric functions associated to root systems. In this paper we define and study an analogue of Jack and Macdonald polynomials for affine root systems. Our approach is based on representation theory of affine Lie algebras and quantum affine algebras, and follows the ideas of our recent papers [EK1], [EK2], [EK3].

We start with a review of the theory of Jack (Jacobi) polynomials associated with the root system of a simple Lie algebra $\mathfrak{g}$. This theory was described in the papers of Heckman and Opdam [HO], [H], [O1], [O2]. In these papers, Jack polynomials are defined as a basis in the space of Weyl group invariant trigonometric polynomials which (1) differs from the basis of orbitsums by a triangular matrix (with respect to the standard partial ordering on dominant integral weights) with ones on the diagonal, and (2) is an eigenbasis for a certain second-order differential operator (the Sutherland-Olshanetsky-Perelomov operator, [Su], [OP]). It turns out that these conditions determine Jack polynomials uniquely. Orbitsums and characters for $\mathfrak{g}$ turn out to be special cases of Jack polynomials. These polynomials have a $q$-deformation—the Macdonald polynomials; they were introduced by I. Macdonald in his papers [M1], [M2] and have been studied intensively since that time.

We generalize the definition of Jack polynomials to the case of affine root systems. We assign such a polynomial to every dominant integral weight of the affine root system. It is done in the same way as for the usual root systems: the only thing one has to do is replace the Sutherland operator by its affine analogue. This analogue is constructed in the same way as for usual root systems, and it turns out to be (after specialization of level) a parabolic differential operator whose coefficients are elliptic functions. This operator was introduced in [EK3] (for the root system $A_{n-1}$) and is closely related to the Sutherland operator with elliptic coefficients considered in [OP], but is more general. Analogously to the finite-dimensional case, orbitsums and characters (of integrable modules) for the affine Lie algebra $\hat{\mathfrak{g}}$ are special cases of affine Jack polynomials.

For orbitsums and characters of affine Lie algebras, there is a beautiful theory of modular invariance described in [K]. We generalize this theory to general affine Jack polynomials. It turns out that the finite-dimensional space spanned by the Jack polynomials of a given level is modular invariant with a certain weight. Moreover, as in the character case, the representation of the modular group in

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