L^p-L^q ESTIMATES FOR FUNCTIONS OF THE LAPLACE-BELTRAMI OPERATOR ON NONCOMPACT SYMMETRIC SPACES. I

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Let G and K be a connected noncompact semisimple Lie group with finite center and a maximal compact subgroup thereof, and consider the symmetric space G/K, also denoted by X. There is a canonical invariant Riemannian metric on X; denote by $-\mathcal{L}_0$ the associated Laplace-Beltrami operator. By general nonsense, \mathcal{L}_0 is positive and essentially self-adjoint on $C_c^{\infty}(X)$; let \mathcal{L} be the unique self-adjoint extension of \mathcal{L}_0 and $\{P_\lambda\}$ the spectral resolution of the identity for which

$$\mathscr{L}f = \int_{b}^{\infty} \lambda \, dP_{\lambda} f \qquad \forall f \in \mathrm{Dom}(\mathscr{L}),$$

where the bottom of the spectrum, b, is equal to $\langle \rho, \rho \rangle$, ρ being the usual half sum of the positive roots. Consider the families of operators $\mathscr{H}_{t,\theta}$, where t > 0 and $0 \leq \theta \leq 1$, and $\mathscr{R}^{\alpha}_{\theta}$ and $\mathscr{S}^{\alpha}_{\theta}$, where $0 \leq \theta \leq 1$ and $\operatorname{Re}(\alpha) \geq 0$, defined by the formulae

$$\begin{aligned} \mathscr{H}_{t,\theta}f &= \int_{b}^{\infty} \exp(-t(\lambda - \theta b)) \, dP_{\lambda} \, f, \\ \mathscr{R}_{\theta}^{\alpha}f &= \int_{b}^{\infty} \, (\lambda - \theta b)^{-\alpha/2} \, dP_{\lambda} \, f, \end{aligned}$$

and

$$\mathscr{S}^{\alpha}_{\theta}f = \int_{b}^{\infty} \left(\lambda - \theta b\right)^{-\alpha/2} \exp(i(\lambda - \theta b)) dP_{\lambda} f,$$

for all L^2 -functions f for which these integrals define L^2 -functions. The purpose of this paper is to study the $L^p - L^q$ boundedness of the families of operators $\mathscr{H}_{t,\theta}, \mathscr{R}^{\alpha}_{\theta}$, and $\mathscr{S}^{\alpha}_{\theta}$, when $1 \leq p, q \leq \infty$.

Over the past few years, considerable effort has been made to understand diffusion processes generated by elliptic operators, and their behaviour for small time may

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