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ARRANGEMENTS OF HYPERPLANES AND VECTOR BUNDLES ON P^n

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Introduction. Let X be a smooth algebraic variety and D be a divisor with normal crossing on X. The pair (X, D) gives rise to a natural sheaf $\Omega^1_X(\log D)$ of differential 1-forms on X with logarithmic poles on D. For each point $x \in X$ the space of sections of this sheaf in a small neighborhood of x is generated over $\mathcal{O}_{X,x}$ by regular 1-forms and by forms $d \log f_i$ where $f_i = 0$ is a local equation of an irreducible component of D containing X. This sheaf (and its exterior powers) was originally introduced by Deligne [De] to define a mixed Hodge structure on the open variety $X - D$. An important feature of the sheaf $\Omega^1_X(\log D)$ is that it is locally free, i.e., can be regarded as a vector bundle on X.

In this paper we concentrate on a very special case when $X = P^n$ is a projective space and $D = H_1 \cup \cdots \cup H_m$ is a union of hyperplanes in general position. It turns out that the corresponding vector bundles are quite interesting from the geometric point of view. It was shown in an earlier paper [K] of the second author that in this case $\Omega^1_{P^n}(\log D)$ defines an embedding of P^n into the Grassmann variety $G(n, m-1)$ whose image becomes, after the Plücker embedding, a Veronese variety V_n^{m-3}, i.e., a variety projectively isomorphic to the image of P^n under the map given by the linear system of all hypersurfaces of degree $m - 3$. In the case when the hyperplanes osculate a rational normal curve in P^n, the bundle $\Omega^1_{P^n}(\log D)$ coincides with the secant bundle E_n^m of Schwarzenberger [Schw1–2]. The corresponding Veronese variety consists in this case of chordal $(n-1)$-dimensional subspaces to a rational normal curve in P^{m-2}.

The main result of this paper (Theorem 7.2) asserts that in the case $m \geqslant 2n + 3$ the arrangement of m hyperplanes $\mathscr{H} = \{H_1, \ldots, H_m\}$ can be uniquely reconstructed from the bundle $E(\mathscr{H}) = \Omega^1_{P^n}(\log \bigcup H_i)$ unless all of its hyperplanes osculate the same rational normal curve of degree n. To prove this we study the variety $C(\mathscr{H})$ of jumping lines for $E(\mathscr{H})$. The consideration of this variety is traditional in the theory of vector bundles on P^n (see [Bar, Hu]). In our case this variety is of some geometric interest. For example, if $n = 2$, i.e., we deal with m lines in P^2, then (in the case of odd m) $C(\mathscr{H})$ is a curve in the dual P^2 containing the points corresponding to lines from $\mathscr{H}$. The whole construction therefore gives a canonical way to draw an algebraic curve through a collection of points in (the dual) P^2.

For 5 points $p_1, \ldots, p_5$ in P^2 this construction gives the unique conic through p_i.

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