

ARITHMETIC AUTOMORPHIC FORMS FOR THE NONHOLOMORPHIC DISCRETE SERIES OF $GSp(2)$

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CONTENTS

Introduction	59
1. Discrete series representations and coherent cohomology classes: a review	63
2. A rationality criterion for $GSp(2)$	66
3. A rationality criterion for $GO(2, 2)$	72
4. The theta correspondence	76
5. The theta correspondence for similitudes	80
6. Seesaw constructions over \mathbb{R}	90
7. The global seesaw identity	100
8. Review and application of some results of Shimura	101
9. Rationality of the theta correspondence for $(GSp(2), GO(2, 2))$	109
References	120

Introduction. Most of the familiar applications to arithmetic of the classical theory of holomorphic modular forms may be traced directly to the existence of a rational structure, derived from algebraic geometry, on the space of such forms. In higher dimensions, it is conjectured that, in most cases, a holomorphic automorphic form shares an L -packet with a collection of nonholomorphic automorphic forms. Most commonly, the archimedean component of every automorphic representation in such an L -packet belongs to the discrete series.

The theory of [11] shows that the set of cuspidal automorphic forms belonging to a given discrete series module also possesses a rational structure derived from algebraic geometry—specifically, from the cohomology of coherent algebraic sheaves—at least when the discrete series parameters are sufficiently regular (“far from the walls”). There is every reason to believe that these rational structures also have broad arithmetic applications. One expects, for example, that all members of the L -packet contribute equally to the arithmetic of their common L -functions. This expectation can be made more precise on the basis of well-known conjectures of Deligne and Langlands (see [12]); suggestive examples from the theory of Hilbert modular forms are described in [9].

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