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DYNAMICAL SYSTEMS AND THE HOMOLOGY NORM OF A 3-MANIFOLD, I: EFFICIENT INTERSECTION OF SURFACES AND FLOWS

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A commonly used technique in the study of manifolds is to take two objects in the manifold and to pull them tight relative to each other by simple operations such as isotopy, until they intersect as efficiently as possible. The properties of efficient intersection are then used to obtain topological information about the manifold.

We shall study the situation where M is a closed, oriented 3-manifold, S is an incompressible surface in M, and $\phi: M \times \mathbb{R} \to M$ is a pseudo-Anosov flow on M. Our main result is the *Efficient intersection theorem*, stated below and with more precision as Theorem 2.0, which says very vaguely that, if ϕ satisfies an extra hypothesis, then S can be isotoped in M so that it intersects the stable and unstable manifolds of ϕ efficiently. The theorem is applied in the sequel [Mos2] to obtain computations of Thurston's norm on $H_2(M; \mathbb{R})$. Following the statement and discussion of the theorem, we give a short historical account of efficient intersection in other contexts in order to motivate the theorem and applications to Thurston's norm. All of the terms in the following discussion are developed in more detail in §§1 and 2.

An Anosov flow ϕ on M is a flow with no stationary points, having 2-dimensional invariant foliations W^s, W^u, such that flow lines in a leaf of the unstable foliation W^u diverge exponentially in forwards time and converge exponentially in backwards time, and flow lines in a leaf of the stable foliation W^s converge exponentially in forwards time and diverge exponentially in backwards time. More generally, ϕ is pseudo-Anosov if it has stable and unstable singular foliations W^s, W^u with the same convergence properties, where W^s, W^u are singular along a collection of finitely many periodic orbits of ϕ, the *singular orbits* of ϕ. If γ is a singular orbit, and if D is a disc transverse to ϕ near a point $x \in \gamma \cap \text{int}(D)$, then $D \cap W^s$ and $D \cap W^u$ are 1-dimensional singular foliations of D, each having a prong singularity at x with negative index. In other words, $D \cap W^s$ and $D \cap W^u$ are locally modelled on the horizontal and vertical foliations of the quadratic differential $z^{2-n} \cdot dz$ on the complex plane $\mathbb{C}$ for some integer $n < 0$ called the *index* of the orbit γ.

Consider a closed, oriented surface $S \subset M$, no component of which is a sphere. Suppose S is incompressible; i.e., for every embedded disc $D \subset M$ with $D \cap S = \partial D$, there is a disc $D' \subset S$ with $\partial D = \partial D'$; equivalently, the induced homomorphism $\pi_1(S) \to \pi_1(M)$ is injective. To state the theorem we must say what it means for S to have efficient intersection with the stable and unstable manifolds. By splitting open

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