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WEIL DIVISORS ON NORMAL SURFACES

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In this note we prepare some basic results to study normal surfaces. We use the intersection theory defined by Mumford [10]. We study the contraction criterion, the projection formula, the Noether formula, the vanishing theorem, etc. We also consider birational morphisms of normal surfaces. We shall introduce a minimal model for a pair of a normal surface and a Q-divisor on it. The author would like to thank the referee for valuable suggestions.

Notation and conventions. A *surface* will mean an irreducible reduced *compact* complex space of dimension 2. A *divisor* will mean a Weil divisor (i.e., a linear combination of irreducible curves) unless otherwise specified. Let Y be a normal surface. We denote by $\mathrm{Div}(Y)$ the group of divisors on Y. An element of $\mathrm{Div}(Y,\mathbf{Q}) = \mathrm{Div}(Y)\otimes\mathbf{Q}$ is called a Q-divisor. For a Q-divisor $D = \sum \alpha_i C_i$ where the C_i are irreducible curves and $\alpha_i \in \mathbf{Q}$ we write as

$$[D] = \sum [\alpha_i] C_i \qquad ([\alpha] \text{ is the greatest integer} \leqslant \alpha)$$

$$\{D\} = \sum \{\alpha_i\} C_i \qquad (\{\alpha\} \text{ is the least integer} \geqslant \alpha).$$

We use "birational morphism" instead of bimeromorphic morphism. A *resolution* is a birational morphism $\pi : X \to Y$ where X is assumed smooth.

1. Intersection theory. Let Y be a normal surface. The intersection pairing $\mathrm{Div}(Y,\mathbf{Q}) \times \mathrm{Div}(Y,\mathbf{Q}) \to \mathbf{Q}$ is defined as follows ([8]). Let $\pi : X \to Y$ be a resolution and let $A = \bigcup E_i$ denote the exceptional set of π. For a Q-divisor D on Y we define the *inverse image* π^*D as

$$\pi^*D = \bar{D} + \sum \alpha_i E_i \tag{1.1}$$

where $\bar{D}$ is the strict transform of D by π and the rational numbers α_i are uniquely determined by the equations: $\bar{D}E_j + \sum \alpha_i E_i E_j = 0$ for all j. Even if D is integral, π^*D is in general a Q-divisor. For two Q-divisors D and D' the *intersection number* DD' is defined to be the rational number $(\pi^*D)(\pi^*D')$.

The following is the normal surface version of the Grauert's contraction criterion theorem.

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