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THE n-WIDTH OF SETS OF ANALYTIC FUNCTIONS

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Introduction. The n-width of a subset A of a Banach space X is defined by

$$d_n(A;X) = \inf_{X_n} \sup_{x \in A} \inf_{y \in X_n} \|x - y\| \tag{1}$$

where X_n runs over *all* n dimensional subspaces of X. The concept of n-width was introduced by Kolmogoroff in [4] and is of central importance in many recent investigations. In this paper we give the exact n-width of some classes of analytic functions and asymptotic values for n-widths in a number of other cases.

In addition to the n-width defined in (1), called the Kolmogoroff n-width, there are two other related concepts. The Gel'fand n-width is defined by

$$d^n(A;X,Y) = \inf_{Y_n} \sup_{x \in A \cap Y_n} \|x\| \tag{2}$$

where Y_n runs over all subspaces of Y of codimension n (Y does not necessarily have to be X). We will call d^n defined in (2) the Gel'fand n-width of A in X relative to Y. The third n-width notion which we will study is the linear n-width defined by

$$s_n(A;X,Y) = \inf_{P_n} \sup_{f \in A} \|f - P_n f\| \tag{3}$$

where P_n runs over all bounded linear operators on Y whose range is a subspace of X of dimension n or less. We shall also determine the Gel'fand and linear n-width for a number of these cases (and show they are equal to the Kolmogoroff n-width as well).

The specific setting of our problem is this. Let Ω be a domain (open, connected set) in the complex plane and let K be a compact subset of Ω. For $1 \leqslant p \leqslant \infty$, let $H_p(\Omega)$ be the Hardy space H_p on Ω, see [9] and let A_p be the restriction to K of the unit ball of $H_p(\Omega)$. Take ν to be a probability measure on K and $1 \leqslant q < \infty$. We will generally use X to represent either $L^q(\nu)$, $1 \leqslant q < \infty$ or $C(K)$ and $\|\cdot\|$ for the usual norm on X. We seek the value of the n-width of A_p in the Banach space X and, in particular, the value of the n-width of A_∞. Throughout the paper, n will be the complex dimension (or codimension) of the subspaces in question.

Section 1 of the paper treats the case when Ω is simply-connected and hence may be taken to be Δ_0, the open disc of radius one centered at the origin. Section

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