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# DECAY OF SOLUTIONS OF SCHROEDINGER EQUATIONS

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**1.** In this paper we develop some simple results which are useful to study the decay of solutions of equations of the type

$$\frac{du}{dt} = iHu, \tag{1}$$

where $H$ is a self-adjoint operator in a Hilbert space. We then apply these results to prove that if $H = \Delta + V$, where $\Delta$ is the self-adjoint Laplace operator in $L^2(\mathbf{R}^3)$ and $V$ is an operator of multiplication by a real valued element of $L^{(3/2)-\epsilon} \cap L^{(3/2)+\epsilon}(0 < \epsilon \leqslant 1/2)$ of suitably small norm, then solutions of (1) with initial data in $L^1$ are of order $|t|^{-3/2}$ for $|t| \to \infty$. This confirms, for $n = 3$, a conjecture made by Strichartz in [5]. We conclude the paper with some remarks on extending this result to the case $n \geqslant 3$.

The following notation will be used throughout. $\mathbf{R}$, $\mathbf{C}$ will denote respectively the field of real numbers, the field of complex numbers. If $z \in \mathbf{C}$, we write $\mathfrak{R}(z)$, $\mathfrak{I}(z)$ to denote the real and imaginary parts of $z$, respectively. If $f$ is a complex valued function on $\mathbf{R}^n$ and if $p \in [1, \infty]$, $\|f\|_p$ denotes the $L^p(\mathbf{R}^n)$-norm of $f$. Integrals in which no region of integration is specified are over the whole space in which the variable of integration is defined. If $\mathcal{H}$ is a Hilbert space, $\mathcal{B}(\mathcal{H})$ denotes the space of bounded operators on $\mathcal{H}$. $\Gamma$ denotes the standard gamma function; i.e.,

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt \qquad \text{for } x > 0.$$

As is usual, by a solution of (1) in the Hilbert space $\mathcal{H}$ we understand a function of the form $u(t) = e^{itH} f$, $f \in \mathcal{H}$.

**2.** Let $\mathcal{H}$ be a complex Hilbert space, $H$ a self-adjoint operator in $\mathcal{H}$. For $\mathfrak{R}(z) \neq 0$, we set $R(z) = (z - iH)^{-1}$. For $j = 1, 2$; let $N_j$ be a map from $\mathcal{H}$ to $[0, \infty]$ such that

a. If $\mathbf{D}_j = \{f \in \mathcal{H} \mid N_j(f) < \infty\}$, then $\mathbf{D}_j$ is a subspace of $\mathcal{H}$ and $N_j$ is a norm in $\mathbf{D}_j$;

b. If $f_n \in \mathbf{D}_j$ for $n = 1, 2, \ldots$; if $f_n \to f$ in $\mathcal{H}$ for $n \to \infty$, and if $\limsup_{n\to\infty} N_j(f_n) < \infty$, then $f \in \mathbf{D}_j$.

THEOREM 1. *Let $C$, $\gamma$ be nonnegative real numbers. The following statements are equivalent*

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