Vol. 46, No. 1 DUKE MATHEMATICAL JOURNAL © March 1979

SINGULARITIES AND ENERGY DECAY IN ACOUSTICAL SCATTERING

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1. Statement of results. Suppose that $\Omega \subset \mathsf{R}^n$, $n \geqslant 2$, is a non-compact connected complete C^∞ submanifold with compact boundary, $\partial\Omega$. In $\mathsf{R}_t \times \Omega$ we consider the wave equation with Dirichlet boundary condition:

$$\begin{aligned} &(\partial_t^2 - \Delta_x)u(t, x) = 0 && \text{in} \quad \mathsf{R}_t \times \Omega \\ &u(t, x) = 0 && \text{on} \quad \mathsf{R}_t \times \partial\Omega \\ &u(0, x) = u_0(x),\ \partial_t u(0, x) = u_1(x) && \text{on} \quad \Omega. \end{aligned} \tag{1.1}$$

In odd dimensions we shall also consider the same problem but with Neumann boundary condition:

$$\begin{aligned} &(\partial_t^2 - \Delta_x)u(t, x) = 0 && \text{in} \quad \mathsf{R}_t \times \Omega \\ &\partial_\nu u(t, x) = 0 && \text{on} \quad \mathsf{R}_t \times \partial\Omega \\ &u(0: x) = u_0(x),\ \partial_t u(0, x) = u_1(x) && \text{on} \quad \Omega. \end{aligned} \tag{1.1$'$}$$

Here, ∂_ν is the inward unit normal vector field at $\partial\Omega$.

The closure in the energy norm

$$E(v, w) = \int_\Omega \left(\sum_{j=1}^n |\partial_{x_j} v(x)|^2 + |w(x)|^2 \right) dx$$

of the space $C_0^\infty(\Omega) \times C_0^\infty(\Omega)$ of pairs of C^∞, complex-valued functions on R^n with compact supports in Ω is a Hilbert space

$$H(\Omega) = \mathring{H}_1(\Omega) \oplus L_2(\Omega)$$

such that if $(u_0, u_1) \in H(\Omega)$ the solution of (1.1) is unique and $\underline{u}(t) = (u(t, \cdot),\ \partial_t u(t, \cdot)) \in H$ for all $t \in \mathsf{R}$. The maps $\underline{u}(0) = (u_0, u_1) \mapsto \underline{u}(t)$ define a unitary group on $H(\Omega)$.

For the solutions to (1.1)$'$ we use the second energy norm:

$$E_2(v, w) = \int_\Omega |\Delta v|^2 + |\nabla v|^2 + |\nabla w|^2 + |w|^2.$$

The closure of the linear space of pairs (v, w) of C^∞ functions on Ω with bounded supports and $\partial_\nu v = \partial_\nu w = 0$ on $\partial\Omega$ defines a Hilbert space $H_N(\Omega)$ on which the solution to (1.1)$'$ provides a unitary group, $U_N(t)$.

Received July 17, 1978.