Vol. 45, No. 1 DUKE MATHEMATICAL JOURNAL© March 1978

SOLUTIONS OF THE KORTEWEG-DE VRIES EQUATION FROM IRREGULAR DATA

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1. Introduction and summary of results

This paper considers the initial value problem

$$u_t - 6uu_x + u_{xxx} = 0 \quad \text{for} \quad t > 0, x \in \mathbb{R}. \tag{1.1}$$

$$u(x, 0) = U(x) \quad \text{for} \quad x \in \mathbb{R}. \tag{1.2}$$

for the Korteweg-de Vries equation. Its primary purpose is to establish the existence and regularity of a solution to (1.1)(1.2) for the "box-shaped" initial function

$$U(x) = H\chi_{[-W,W]}(x); \qquad H \neq 0, \ W > 0. \tag{1.3}$$

For future reference this will be called Problem I. Such a U belongs to $L^2(\mathbb{R})$, but not to H^s for any $s \geq 1$. Thus this problem falls outside the scope of the existence theorems of Bona and Smith [4], Bona and Scott [3], and Saut and Temam [8]. The following theorem will be proved in Section 4.

THEOREM 1.1. *There is a function $u = u(x, t)$ of class C^∞ in $\{(x, t) : x \in \mathbb{R}, t > 0\}$ which solves the Korteweg-de Vries equation* (1.1) *in the region $t > 0$ and which satisfies the initial condition* (1.2)(1.3) *in the weak sense that, for any interval $[a, b]$,*

$$\lim_{t \downarrow 0} \int_a^b u(x, t)dx = \int_a^b U(x)dx.$$

For any fixed $t > 0$, all derivatives $\partial_x^j u(x, t)$ decay rapidly to zero as $x \to +\infty$.

Theorem 1.1 remains valid for any compactly supported step function U in (1.2).

The existence of a C^∞ solution to Problem I is of interest in light of numerical experiments which obtained from a box-shaped initial function wildly oscillatory behavior suggesting the possibility of a discontinuous solution [12]. Such oscillatory behavior is, however, probably intrinsic to Problem I and not the result of numerical instability. It is plausible that just after time $t = 0$ the solution to Problem I is very close to the solution of the linearized equation $v_t + v_{xxx} = 0$ with $v(x, 0) = U(x)$. In Section 5 we see that this $v(x, t)$ is indeed highly oscillatory as t decreases to 0 with $x < W$. Further this $v(x, t)$ approach-

Received September 9, 1977.